Structural Reliability
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Lecture –178
Capacity Demand Component Reliability (Part 26)

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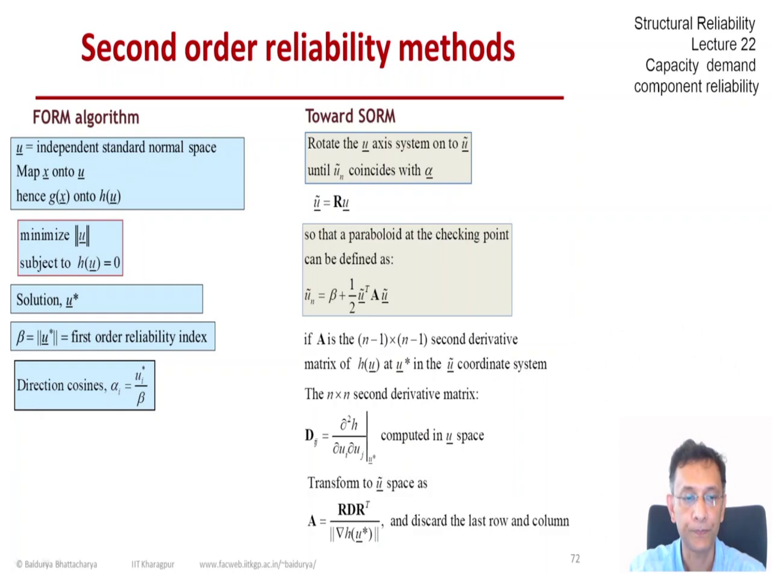

So here is one way we could derive such a second order curve such a parabola and then how to interpret that to improve our reliability estimate. So, the FORM algorithm just for completeness let us put here we find u star the minimum distance point and the distance beta. We are going to use these we are going to use u star and we are going to use this distance beta to develop the second order approach. We will also need the direction cosines that the minimum distance point makes with the origin.

So, the first step is to rotate our independent standard normal axis system. So, let us say we rotate the u-axis system to the u-tilde axis system and the purpose of doing that is to have a a simpler description of the second order parabola paraboloid that we want to fit at the minimum distance point obtained from FORM. So, we stick to the minimum distance point that is the answer given by FORM. So, let us say we take one of the axis the last one u tilde n to coincide with the that minimum distance direction which is given by the direction cosines alpha which we just stated on the left part of your screen.

So, that is the formal description of the transformation the u tilde is R times u R is the transformation matrix. So, if we do it this way then our paraboloid can be given in terms of the nth axis the last one in the new system in the new tilde system is as beta minimum distance plus a second order fit at the minimum distance point u star in the new coordinate system u tilde. So, we need the matrix of second derivatives A.

And so, how do we get that let us let us do that. So, A is an n by n, n - 1 by n - 1 order matrix at defined at the point u star and if we know what the second derivative n by n second derivative matrix is of the limit state function h at u star in the original u space then we can derive A in terms of D using the transformation matrix. So, D is the n by n second derivative matrix of h evaluated at u star and using the transformation R that we already have we get a in terms of the D and because A is n - 1 by n - 1 we discard the last row and the last column obtained this way.

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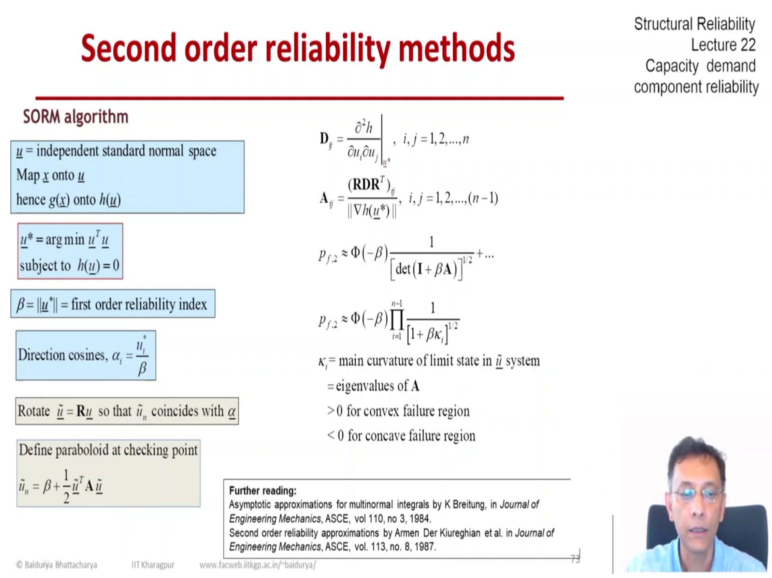

So, to summarize we find the minimum distance point in the independent standard normal space just as we did in form. So, that would be u star we find the beta of the minimum distance we find the direction cosines that this minimum distance makes with the origin and then we try to fit a paraboloid at that minimum distance point. So, we are basically going towards finding the curvature of this paraboloid.

So, we transform u to u tilde and we know how to get that n by n - 1 square matrix A because we can compute D and then from there we can get A. So, these are the mathematical steps and then it can be shown and I have the references at the end of this slide is the improved failure probability would be the FORM failure probability phi of minus beta times a correction factor.

This correction factor takes into account the curvature of the limit state. So, sometimes it could be more than one sometimes it could be less than one depending on whether your failure region is concave or convex depending on whether you are under counting or over counting failure probability. Obviously it cannot give the exact answer but it does make an improvement as we will see by solving the same problems we did before and resolving them with SORM.

So, we can express this if we just take the first term in that expansion we end up with the principal curvatures of that paraboloid which we fit in the u-tilde system if we know the eigen values of the matrix A and the curvature is positive for convex failure region because we need to we need to reduce the failure probability. So, the denominator has to be more than 1 if P f 2 has to be less than phi of minus beta and kappa has to be negative if we have to if we have a concave failure region in which case we need to increase the failure probability obtained by FORM.

So, that is an intuitive explanation of what is going on as I said for further reading I refer you to these very important two papers the one by routing coming out in 1984 and the other by professor Dirk Huragan et al coming out in 1986.