

Structural Reliability
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Lecture –178
Capacity Demand Component Reliability (Part 26)

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Second order reliability methods

FORM algorithm

\underline{u} = independent standard normal space
Map \underline{x} onto \underline{u}
hence $g(\underline{x})$ onto $h(\underline{u})$

minimize $\|\underline{u}\|$
subject to $h(\underline{u}) = 0$

Solution, \underline{u}^*

$\beta = \|\underline{u}^*\|$ = first order reliability index

Direction cosines, $\alpha_i = \frac{u_i^*}{\beta}$

Toward SORM

Rotate the \underline{u} axis system on to $\underline{\tilde{u}}$
until \tilde{u}_n coincides with \underline{e}

$\underline{\tilde{u}} = \mathbf{R}\underline{u}$

so that a paraboloid at the checking point
can be defined as:

$\tilde{u}_n = \beta + \frac{1}{2}\underline{\tilde{u}}^T \mathbf{A} \underline{\tilde{u}}$

if \mathbf{A} is the $(n-1) \times (n-1)$ second derivative
matrix of $h(\underline{u})$ at \underline{u}^* in the $\underline{\tilde{u}}$ coordinate system

The $n \times n$ second derivative matrix:

$\mathbf{D}_i = \frac{\partial^2 h}{\partial u_i \partial u_i} \Big|_{\underline{u}^*}$ computed in \underline{u} space

Transform to $\underline{\tilde{u}}$ space as

$\mathbf{A} = \frac{\mathbf{RDR}^T}{\|\nabla h(\underline{u}^*)\|}$, and discard the last row and column

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So here is one way we could derive such a second order curve such a parabola and then how to interpret that to improve our reliability estimate. So, the FORM algorithm just for completeness let us put here we find \underline{u}^* the minimum distance point and the distance β . We are going to use these we are going to use \underline{u}^* and we are going to use this distance β to develop the second order approach. We will also need the direction cosines that the minimum distance point makes with the origin.

So, the first step is to rotate our independent standard normal axis system. So, let us say we rotate the \underline{u} -axis system to the $\underline{\tilde{u}}$ -axis system and the purpose of doing that is to have a simpler description of the second order parabola paraboloid that we want to fit at the minimum distance point obtained from FORM. So, we stick to the minimum distance point that is the answer given by FORM. So, let us say we take one of the axis the last one $\underline{\tilde{u}}_n$ to coincide

with the that minimum distance direction which is given by the direction cosines alpha which we just stated on the left part of your screen.

So, that is the formal description of the transformation the \tilde{u} is R times u R is the transformation matrix. So, if we do it this way then our paraboloid can be given in terms of the n th axis the last one in the new system in the new \tilde{u} system is as beta minimum distance plus a second order fit at the minimum distance point u^* in the new coordinate system \tilde{u} . So, we need the matrix of second derivatives A .

And so, how do we get that let us let us do that. So, A is an n by n , $n - 1$ by $n - 1$ order matrix at defined at the point u^* and if we know what the second derivative n by n second derivative matrix is of the limit state function h at u^* in the original u space then we can derive A in terms of D using the transformation matrix. So, D is the n by n second derivative matrix of h evaluated at u^* and using the transformation R that we already have we get A in terms of the D and because A is $n - 1$ by $n - 1$ we discard the last row and the last column obtained this way.

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Second order reliability methods

SORM algorithm

\tilde{u} = independent standard normal space
Map \tilde{x} onto \tilde{u}
hence $g(\tilde{x})$ onto $h(\tilde{u})$

$\tilde{u}^* = \arg \min_{\tilde{u}} \tilde{u}^T \tilde{u}$
subject to $h(\tilde{u}) = 0$

$\beta = \|\tilde{u}^*\|$ = first order reliability index

Direction cosines, $\alpha_i = \frac{\tilde{u}_i^*}{\beta}$

Rotate $\tilde{u} = R\tilde{u}$ so that \tilde{u}_n coincides with α

Define paraboloid at checking point

$\tilde{u}_n = \beta + \frac{1}{2} \tilde{u}^T A \tilde{u}$

$D_{ij} = \frac{\partial^2 h}{\partial u_i \partial u_j} \Big|_{u^*}, i, j = 1, 2, \dots, n$

$A_{ij} = \frac{(RDR^T)_{ij}}{\|\nabla h(u^*)\|^2}, i, j = 1, 2, \dots, (n-1)$


$p_{r,2} \approx \Phi(-\beta) \frac{1}{[\det(I + \beta A)]^{1/2}} + \dots$

$p_{r,2} \approx \Phi(-\beta) \prod_{i=1}^{n-1} \frac{1}{[1 + \beta \kappa_i]^{1/2}}$

κ_i = main curvature of limit state in \tilde{u} system
= eigenvalues of A
> 0 for convex failure region
< 0 for concave failure region

Further reading:
Asymptotic approximations for multinomial integrals by K Breitung, in *Journal of Engineering Mechanics*, ASCE, vol 110, no 3, 1984.
Second order reliability approximations by Armen Der Kiureghian et al. in *Journal of Engineering Mechanics*, ASCE, vol. 113, no. 8, 1987.

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So, to summarize we find the minimum distance point in the independent standard normal space just as we did in form. So, that would be u^* we find the beta of the minimum distance we find the direction cosines that this minimum distance makes with the origin and then we try to fit a

paraboloid at that minimum distance point. So, we are basically going towards finding the curvature of this paraboloid.

So, we transform u to u tilde and we know how to get that n by $n - 1$ square matrix A because we can compute D and then from there we can get A . So, these are the mathematical steps and then it can be shown and I have the references at the end of this slide is the improved failure probability would be the FORM failure probability ϕ of minus beta times a correction factor.

This correction factor takes into account the curvature of the limit state. So, sometimes it could be more than one sometimes it could be less than one depending on whether your failure region is concave or convex depending on whether you are under counting or over counting failure probability. Obviously it cannot give the exact answer but it does make an improvement as we will see by solving the same problems we did before and resolving them with SORM.

So, we can express this if we just take the first term in that expansion we end up with the principal curvatures of that paraboloid which we fit in the u -tilde system if we know the eigen values of the matrix A and the curvature is positive for convex failure region because we need to reduce the failure probability. So, the denominator has to be more than 1 if $P f 2$ has to be less than ϕ of minus beta and κ has to be negative if we have to if we have a concave failure region in which case we need to increase the failure probability obtained by FORM.

So, that is an intuitive explanation of what is going on as I said for further reading I refer you to these very important two papers the one by routing coming out in 1984 and the other by professor Dirk Huragan et al coming out in 1986.