

Structural Reliability
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Lecture –177
Capacity Demand Component Reliability (Part 25)

When solving reliability problems with FORM earlier in this lecture and in the previous one we saw again and again that FORM was giving approximate answers and the basis of our comparison was Monte Carlo simulations for the same problems and which presumably was a very accurate answer I mean the Monte Carlo results provided the sampling was done correctly and the sample size was very, very large.

And we know why this error this approximation deviation happens we discussed that one is caused by the loss of information potential loss of information during the map from the x space the basic variable space onto the standard the independent standard normal space u . And the second place this happens is when we linearize this limit state function in the independent standard normal space.

The first problem or the first issue we could address by using a better map which preserves more probabilistic information instead of Hasofer-Lind map for example we could use a full distribution transformation if correlation information was given we used Nataf transformation if all distribution information is given then we could use a Rosenblatt transformation. So we did employ those in the examples the full distribution and the Nataf type.

In the second part where we linearize the limit state in the independent standard normal space could we do something if the limit state is not a straight line not a linear function of the use can we do something to reduce that error. So that is actually the motivation behind SORM the Second Order Reliability Method and that is what we are going to discuss next.

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Second order reliability methods

Recap: FORM

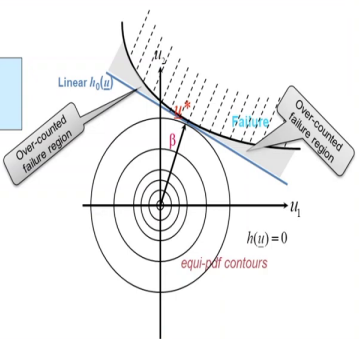
\underline{u} = independent standard normal space
Map \underline{x} onto \underline{u}
hence $g(\underline{x})$ onto $h(\underline{u})$

minimize $\|\underline{u}\|$
subject to $h(\underline{u}) = 0$

Solution, \underline{u}^*

$\beta = \|\underline{u}^*\|$ = reliability index

$$P_f \approx \Phi(-\beta)$$



Is there a way to reduce the error caused by linearization?



So let us recap what we do in FORM we map from x onto u and then g to h and we minimize the distance and that is what you see u^* is the minimum distance point and the distance is β and we say that the failure probability is $\Phi(-\beta)$ approximately. Now and we saw in our examples that we were under counting the failure probability because we were ignoring those shaded regions that I just introduced in the in the plot. So, on both sides you have under counted failure regions.

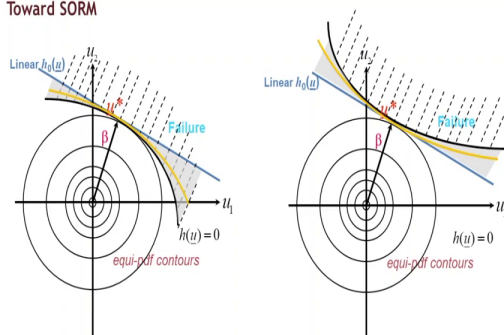
So those failure probabilities are not showing up in in our answer of $\Phi(-\beta)$ obviously the failure region doesn't have to be concave like this it could be convex and it would then if we linearized h with h_0 then we would over count we would be over counting the failure probability because now we have those additional shaded regions which actually do not contribute to failure in reality.

So, the question is that can we reduce this error so that is what we already mentioned at the beginning of this section.

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Second order reliability methods

Toward SORM



Can we fit a higher order curve at u^* instead of the first order straight line?

How about a second order parabola (curved the same way)?



And so putting all of this together we have either a convex or a concave failure region and we have the ability to find the minimum distance but when we linearize and which we must do to estimate the failure probability we either end up under counting or over counting. So that is that is the state. So now the question naturally arises is that you know we are using the first order that is why the first order term is used in the method FORM.

Because we use a first order or a linear approximation can we use a higher order curve instead of that straight line? So what about we use just next higher order the second order so what we use parabola curved in the same sense obviously. So if we use the orange lines to be a better approximation to the black line compared to the blue line so the black line is the actual limit state the blue line was our linear approximation.

Now somewhere in between can we do that or when it is convex then that is what it would look like and then what would be the failure probability the improved failure probability in that case. So how do we get that orange line and then how do we compute the improved reliability. So that is the subject of today's subject of this section's discussion.