Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –176
Capacity Demand Component Reliability (Part 24)

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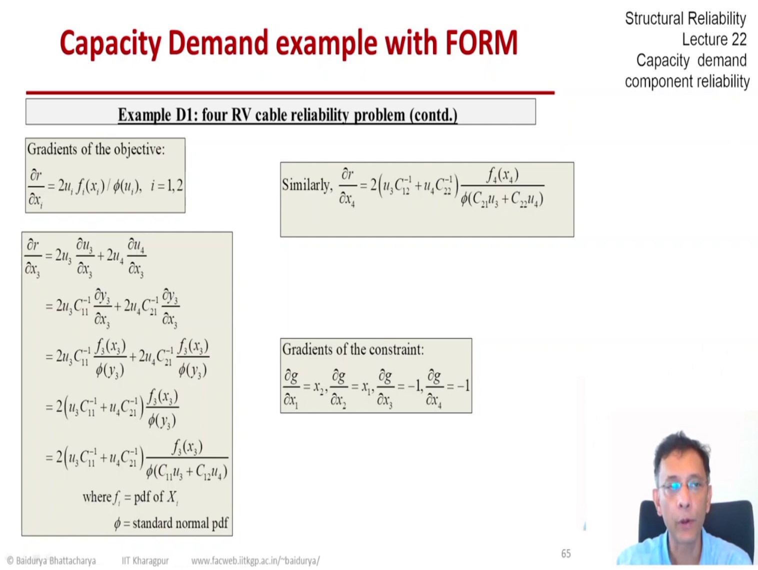

If we take a gradient based approach to solve this optimization problem we would need to provide the gradients of the objective function. So, let us see what those gradients would look like. For the first two random variables x 1 and x 2 they are straightforward we have already looked at them in problem C1 earlier in this lecture. So, the partial of r with respect to x 1 r being the objective function and x 1 is the first decision variable that would be twice u1 times the pdf of x 1 over the normal standard normal pdf of u1.

How does that come we use the chain rule of differentiation. So, we first take the derivative of r in terms of u 1 and then u 1 with respect to x 1 there we employ the full distribution transformation relation between u 1 and x 1. So, phi capital phi of u 1 is equal to capital f of x 1. So, partial derivative of that with respect to x 1 would give me small phi of u 1 times partial u1 partial x 1 and on the right hand side it is the pdf of x 1.

So that gives me this final form as twice u 1 f of x 1 over phi of u 1 and this is also valid for x 2 and u 2 because they are mutually independent of all the rest. Now for x 3 and x 4 we need to take into account that there is a dependent structure. So, let us proceed step by step. So, partial of r with respect to x 3 is now involves both u 3 and u 4 because they are related as we saw in the previous slide. So, partial u 3 with respect to x 3 and partial u 4 with respect to x 3.

There would be some terms which do not count and sum which do because u 3 and u 4 we can now write in terms of the y's the y 3 and y 4 those intermediate variables. So, we see that the first set u partial u 3 with respect to partial x 3 that has the y 3 terms coming from the linear relationship we saw in the previous slide and the second term partial u 4 with respect to x 3 that also has a partial y 3 contribution. So, there now we have partial y 3 with respect to x 3 occurring twice.

And just like we did for x 1 and x 2 the partial of y with respect to the x we will use the same full distribution transformation information and that gives me the ratio of the density functions. So, the pdf of x 3 over the pdf of y 3 y 3 and y 4 are both standard normal and that let us express the partial of r with respect to x 3 in terms of C 11 inverse C 21 inverse u 3 and u 4 and the ratio of the two density functions and if we want to express y 3 in terms of u 3 and u 4.

We can do that and that closes the entire expression. So, we now have partial of r with respect to x 3 in terms of u 3 u 4 and the density function of x 3 likewise we can derive a similar expression for the partial of r with respect to x 4 there we have the density of x 4 and again u 3 and u 4 coming together the gradients of the constraint if they're useful they are very simple because the the constraint is a very simple function of x 1 x 2 x 3 and x 4.

And if we again employ the MATLAB program for example f main con that we used for program for problem C1 we would get the solution.

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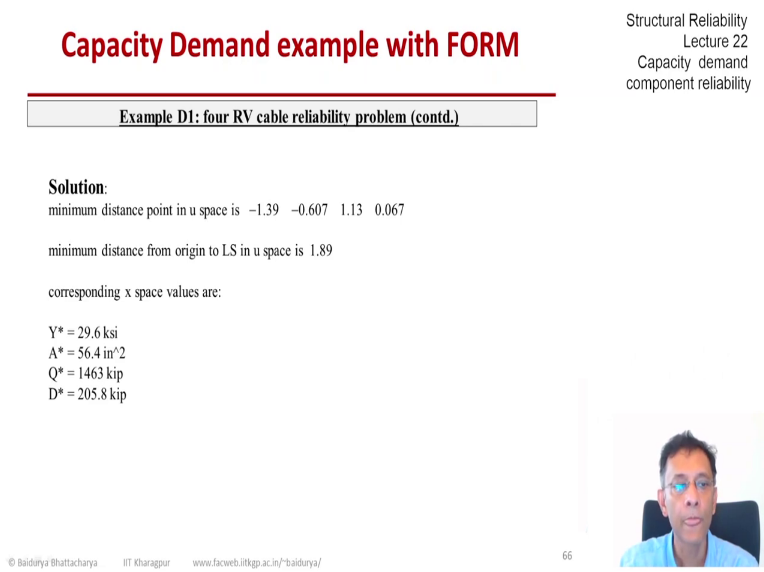

And I m just presenting the solution here if you would like you may want to write the code yourself and compare the results with what you see here the minimum distance points a new space for the yield is about 1.4 times the standard deviation below the mean for the cross-sectional area it is also about 60 standard deviation below the mean for the loads they are above the mean the dead load is almost at the mean.

So the minimum distance point beta the reliability index is 1.89 and the corresponding design values the checking point values in the basic variable spaces what you see on the screen about 30 ksi for yield about 56 square inch for area about 1463 kip for the live load which is less than what we saw in problem C1 and the dead load is slightly above the mean at about 206 kpi. Now we might be interested to know and this is obviously a good question as to what would happen if we did not take the trouble of going through this nataf transformation.

We did that because we wanted to be true to the dependence information that was provided that correlation coefficient between D and Q.

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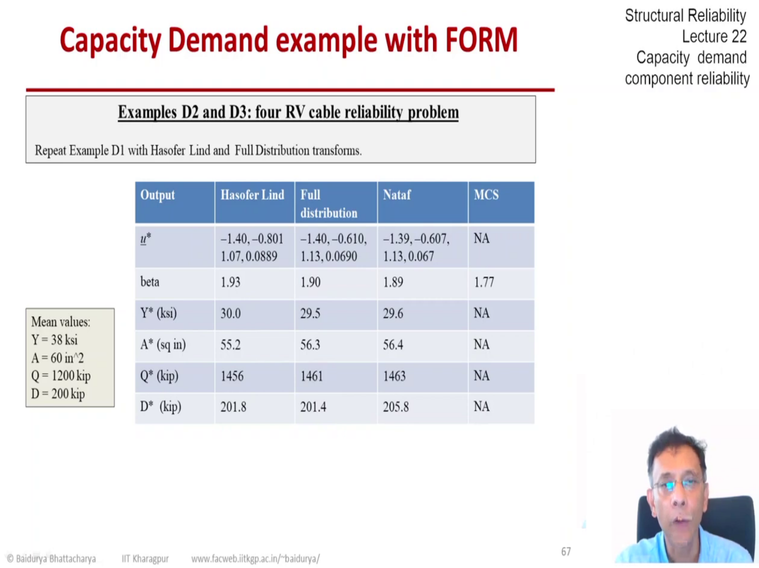

Output	Hasofer Lind	Full distribution	Nataf	MCS
$\underline{u}^*$	–1.40, –0.801 1.07, 0.0889	–1.40, –0.610, 1.13, 0.0690	–1.39, –0.607, 1.13, 0.067	NA
beta	1.93	1.90	1.89	1.77
Y* (ksi)	30.0	29.5	29.6	NA
A* (sq in)	55.2	56.3	56.4	NA
Q* (kip)	1456	1461	1463	NA
D* (kip)	201.8	201.4	205.8	NA

But what if we employed naively the Hasofer-Lind or the full distribution transformation where in the first case the Hasofer-Lind is not even able to preserve all the probabilistic information in the transformation and the full distribution while it can preserve the probability information for each variable it cannot incorporate any dependence information. So, here would be the results our our beta comes to about 1.93 what we had before in the nataf was 1.89.

So, just a little bit over estimation of the reliability the design values are roughly very similar as we were as we stated in the previous slide if you compare to the mean values you see that Y and A are below the mean D and Q are above the mean d is slightly above the mean if we went one step better from Hasofer-Lind we employed the full distribution type transformation then the result hardly changes beta comes down from 1.93 to 1.90.

So, it gets closer to the nataf result the design values are roughly around the same in the same region and just to recap what we got for the nataf was a beta 1.89 and we have already discussed the design point values. Now how do they compare with the true value if we can find the true value and hoping that Monte Carlo simulations give the right result backed by the law of large numbers.

We have also solved this by Monte Carlo simulations and the result is a slightly lower reliability at 1.77. So, if that 1.77 value is right for reliability index then nataf is definitely closer to that compared to Hasofer-Lind and full distribution but this was a simple problem the limit state was very well behaved it did not have an inflection point it did not have the curvature changing drastically. So, we see that the three transformations have a full lane full distribution and net of roughly giving very, very close results.

Now the Monte Carlo simulation is lower than that gives a lower reliability. So, again here form overestimated reliability one final point to note and this is in favor of FORM type approaches is that the Monte Carlo simulation can give us an equivalent beta but it cannot give us the design point. So, if we wanted kind of intuitive understanding of what are the most likely points or what sort of design value we should aim for then Monte Carlo simulation directly will not be able to provide that information.