Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –175
Capacity Demand Component Reliability (Part 23)

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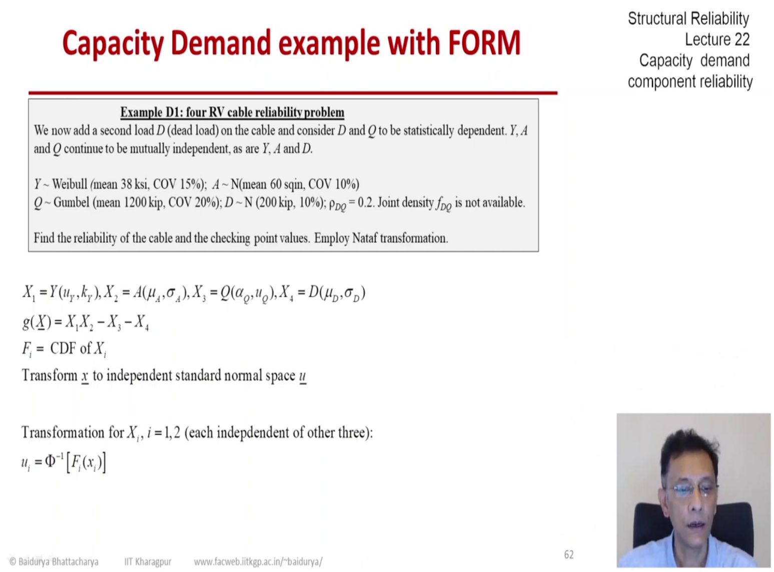

With this example D1 we now move on to a four random variable problem which we will solve with FORM. We continue with the same problem set up that we used in problem groups B and C that there is a cable in tension but there is an important twist here as well. So, let us first read this and then we will talk about that question. So, as you see we have added fourth random variable a dead load in addition to the yield strength of the cable the cross-sectional area of the cable the live load Q and a dead load D.

The fourth one the new one now Y, A and Q they are distributed as before D is normal but importantly and that is the twist D and Q are dependent and the dependence information is not complete we have been given partial information and which is actually quite typical is in addition to knowing the marginal distributions. We know the correlation coefficient the joint density is not available.

So, we just are not able to employ for example the Rosenblatt transformation which would be possible if we knew the complete dependent structure but we have knowledge of the correlation coefficient rho between D and Q. So, obviously if we have to take that into account we one of the clear options is the Nataf transformation which is what has been recommended in this problem statement. So, let us tackle this.

This is these are the four random variables X 1 X 2 X 2 and X 4 the limit state now is X 1 X 2 minus X 3 minus X 4 now how do we do the transformation for the independent random variables there is not any difference really from what we did in the previous example C1 when they were all independent. So, let F be the cdf of x. So, I going from one to four now how do we transform for 1 and 2 which is yield and area. We will just do the full distribution transformation as we did before nothing has changed.

So, you can go back to those steps and see how you did that for X 1 and X 2 things become different and interesting for X 3 and X 4 and let us take that up.

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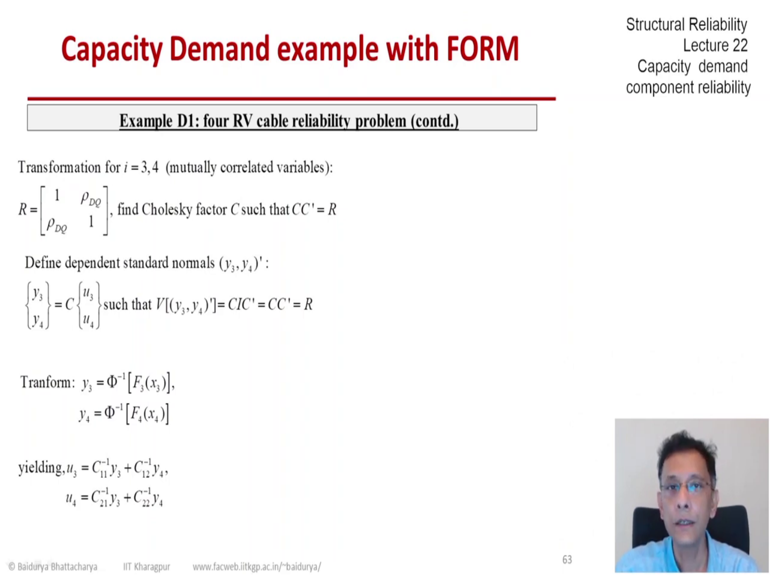

So, in for X 3 and X 4 this is the correlation coefficient rho dq and this is the correlation matrix. So, one in the diagonals and rho dq in the of diagonal positions. So, as we discussed when we were discussing Nataf transformation earlier. In the previous lecture we find that Cholesky factor C of R. So, that C, C transpose is equal to R, R being the correlation matrix of D and Q or X 3 and X 4.

Now we define dependent standard normal variables in terms of the independent standard normal. So, X 3 goes to Y 3 goes to u 3 X 4 goes to Y 4 goes to u 4. So, you remember that map. So, let us talk about the second and the third in this sequence. So, Y 3 and Y 4 are given in terms of a linear combination of u 3 and u 4 and because the way we have defined c obviously the even though the u's are independent Y 3 and Y 4 have a correlation coefficient of r that is what we intended to do.

So, we have achieved that now let us employ the full distribution transformation between the Y's and X's and thereby we are imposing the dependence between X 3 and X 4 as well. This is how we transform Y 3 to X 3 and Y 4 to X 4 and just for closing the loop u 3 is a linear combination of the of the Y’s and u 4 is another linear combination of the Y’s. So, this would let us express the X's the X 3 and X 4 in terms of u 3 and u 4 and because the way we have defined Y 3 and Y 4, X 3 and X 4 would be correlated.

This one subtle point here is that the correlation coefficient between X 3 and X 4 achieved this way would probably be slightly different from rho dq because of the non-linear transformations involved between Y and X in most cases this is small enough but if we want to be very accurate then we would need to do an iterative solution to pick the exact value of rho dq in finding C the matrix C so, that we are able to impose the exact rho dq between X 3 and X 4.

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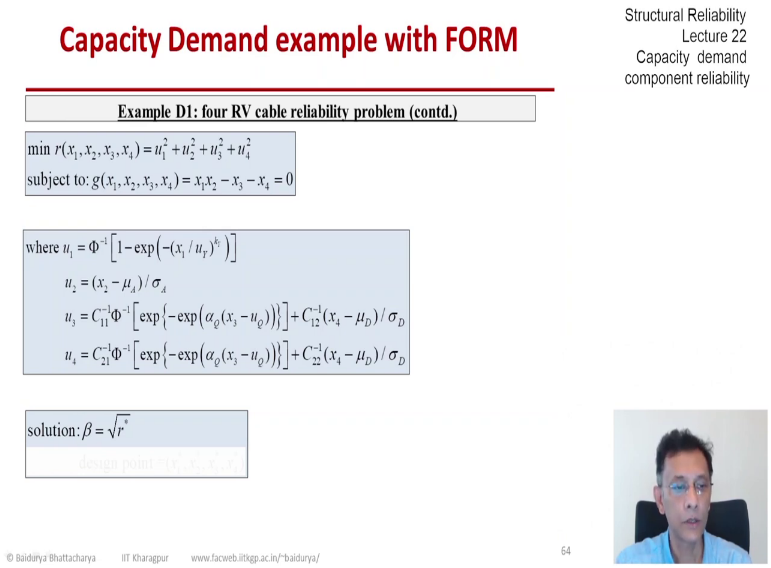

We are now ready to set up the optimization problem. So, and as we did for C 1 example C 1 we will stay confined to the X space. So, our decision variables would be X 1 X 2 X 2 and X 4 but we have to minimize the distance in u space. So, the way we achieve that is define an objective function R in terms of the square of the use and the constraint is of an equality type that g of X 1 X 2 X 3 X 4 is equal to 0 which is X 1 X 2 minus X 3 minus X 4 equals 0. And we need to be able to express the use in terms of the axis.

So we have been doing this in the last in the previous two slides. So, let us just put everything together u1 is related to X 1 through the full distribution transformation which you see on the screen phi of u 1 is the F of the steel strength which is the viable form for u 2 and X 2 it is simple normal to normal transformation and then for X 2 and X 4 to u 3 and u 4 we need to linearly combine them after doing a nonlinear transformation which we just saw in the previous slide.

So, putting all of that together u 3 is C 11 inverse of the distributional transformation with X 3 plus C 12 inverse with the distribution transformation of X 4, X 4 being normal you see the simple X 4 minus mu D over sigma default there. Likewise for u 4 we express in terms of X 3 and X 4 the same way with coefficients C 21 inverse and C 22 inverse. So, the problem setup is now complete we have an objective function R as you see on the top of your screen.

The constraint is given just below that and now the hidden relation between the X's and the user all stated in the box below. The solution is the square root of the optimum R star and the design point if we want would be the output of the optimization exercise.