

Structural Reliability
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Lecture –174
Capacity Demand Component Reliability (Part 22)

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Capacity Demand example with FORM

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Example C1: three RV cable reliability problem (contd.)

```
function main
clear clear all; close all;
global uQ alphaQ muA sdA uY kY
global z1 z2 z3
%basic variable statistics
muQ=1200;VQ=2;sdQ=muQ*VQ;alphaQ=pi/sqrt(6)/sdQ;uQ=muQ-0.5772/alphaQ; % Q is Gumbel, kips
muA=60;VA=0.1;sdA=muA*VA; % A is Normal in2
muY=38;VY=0.15;
for k=1:0.01:100 %finding shape parameter for Weibull
    v2=gamma(1+2/k)/(gamma(1+1/k))^2-1; vs=sqrt(v2);
    if(vs<VY) kY=k; break; end
end
uY=muY/gamma(1+1/kY); % scale parameter for Weibull, ksi
% setting up call to optimization
A = []; b = []; Aeq = []; beq = []; lb = []; ub = [];
nonlcon=@formconstraint;
x0=[muY muA muQ]'; % initial values, note in which order the variables are listed
options = optimoptions('fmincon','display','final-detailed','SpecifyObjectiveGradient',true)
[x,fval,exitflag,output] = fmincon(@betasquare,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)
beta=sqrt(fval);
fprintf(1,'minimum distance point in z space is %f %f %f \n',z1,z2,z3);
fprintf(1,'minimum distance is %f \n',beta);
Y=x(1);Acable=x(2);Q=x(3); %recovering design point basic variables
fprintf(1,'corresponding x space values are Y=%f.2f(ksi) A=%f.2f(in^2) Q=%f.2f(ksi) \n',Y,Acable,Q)
end
```



Now let us go through the steps we used to solve this optimization problem for problem C1 and as I said I have solved it using MATLAB and so, let us go through the code and this will help you write your own code for other problems as well. So, we define the main function where all the variables are either defined or computed and the main function is called the f main con which is one of the standard programs in MATLAB for constrained non-linear optimization it is a gradient-based algorithm.

So, if available gradient should be given to speed up the process and for greater accuracy. So, we are going to give those as well. But let us see the steps what you see on the first few lines is I have shared certain variables they will be useful to transfer between the main program and all the functions or subroutines. The first global just deals with the distribution parameters for Q and A and Y.

The second one those z_1 , z_2 and z_3 are basically the independent standard normal variables I have used z or z instead of u partly because I did not want to confuse the u of the variable with this u of standard normal. So, I hope he will not get confused. So, the z s or the z 's here are the independent standard normal variables. So, in the next block I compute all the statistics from the given information and the mean and standard deviation of A the u and k of the viable Y .

And I think the gumball is also defined there in the top. Now we start the block of setting up the optimization this line that you see are some default arrays and matrices that have to be defined in terms of constraints for that F main con function we will define the nonlinear constraint which would be given in function form constraint. So, that is you are going to see it in the next slide. We have to define the initial points which I choose to define as as the mean the mean vector the next line are the various options in the f main con. One of the things is that I have the objective gradients. So, I am going to provide them.

And then I call the function f main con and then that nonlinear constraint has been provided the objective function is beta square. So, that is something also we are going to show in the next slide. Then we once f main con does its job hopefully it will converge and then we will get the answer beta would be the square root of f val because your objective is basically u_1 squared plus u_2 squared plus u_3 square or z_1 squared plus z_2 squared plus z_3 square in this program's terminology and then we output the results.

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Example C1: three RV cable reliability problem (contd.)

```
function [u2 delu2]=betasquare(x)
global uQ alphaQ muA sdA uY KY
global z1 z2 z3

Y=x(1); Acable=x(2); Q=x(3);
FY1=exp(-(Y/uY)^KY); z1=norminv(FY1);
z2= (Acable-muA)/sdA;
FQ= exp(-exp(-alphaQ*(Q-uQ))); z3=norminv(FQ);

u2=1^2+z2^2+z3^2;
% objective function is squared distance to LS
in z space
% we need del obj/ del x = del obj/ del z * del
z/ del x
% del z/del x is obtained from above definition
of each z in terms of corresponding x
fY=exp(-(Y/uY)^KY)*KY*(1/uY)^(KY-1)/Y;
delu2(1)=2*z1*fY/normpdf(z1);
delu2(2)=2*z2/sdA;
FQ= FQ^alphaQ*exp(-alphaQ*(Q-uQ));
delu2(3)=2*z3*FQ/normpdf(z3);
end
```

```
function [c,ceq] = formconstraint(x)
global uQ alphaQ muA sdA uY KY
global z1 z2 z3
Y=x(1); Acable=x(2); Q=x(3);
ceq = Y*Acable-Q;
c = [];
end
```



So, let us go just take a look at the two functions that I mentioned here. So, the first function is the objective function. So, it not only gives the value of the objective but also the gradients here I take advantage of the global variables all the u's and alphas and the z 1 z 2 and z 3. So I rename the x's to Y area of the cable and Q just for my convenience and then I compute the cdfs because I am going to need them in defining the sets because remember it is a full distribution transformation that we are employing.

So, I would have to invert distribution to distribution. So, once for Y I get the cdf then I can do the normal inverse. So, z 1 is norm of fy. So, there I am calling another function a library function in MATLAB which inverts the normal cdf z 2 is easy because it is a normal to normal transformation and likewise for Q I get set 3's through the normal inverse. And then u 2 the objective function is the square of the distance.

So, z 1 square plus z 2 square plus X squared. So, I have been true to the problem set up. So, far now as additional information which would help the optimization I would provide the gradients because I know because I have the ability. So, as I said the partial of the objective with all the x's X 1 X 2 X 3 I am going to use chain rule of differentiation and they would be given as you see here I obtain first of all the density function of Y and in terms of the chain rule del u which would be the objectives.

Objective in terms of X 1 that partial derivative is 2, z 1 times the ratio of the density functions and likewise for u 2 or z 2 and z 3. Once I am done I send all this information back to the main program and then it continues as many times as this is called the next block is the constrain function and since we are operating in the X space our constraint is very simple looking. So, it is basically Y times a minus Q which is what you see and we at this point we are not giving the gradients of the constraint function. So, that is the program behind the optimization, optimization for problem C1.

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Example C1: three RV cable reliability problem (contd.)

Optimization completed: The relative first-order optimality measure, 5.521154e-07, is less than options.OptimalityTolerance = 1.000000e-06, and the relative maximum constraint violation, 2.105312e-16, is less than options.ConstraintTolerance = 1.000000e-06.

Optimization Metric Options
relative first-order optimality = 5.52e-07 OptimalityTolerance = 1e-06 (default)
relative max(constraint violation) = 2.11e-16 ConstraintTolerance = 1e-06 (default)

output =
iterations: 21
funcCount: 107
constrviolation: 2.2737e-13
stepsize: 0.0048
algorithm: 'interior-point'
firstorderopt: 5.5212e-07
cgiterations: 1

x =
1.0e+03 *
0.0279
0.0561
1.5652

fval = 5.0938
exitflag = 1

minimum distance point in z space is -1.620935-0.653822
1.427895
minimum distance is 2.256944
corresponding x space values are Y=27.91(ksi)
A=56.08(in^2) Q=1565.19(ksi)



And here is the output from the MATLAB code the program terminated successfully and here are the tolerances mentioned the default tolerances that the program used and these are some further details and here is the output the optimal output in terms of the X 's the decision variables the objective function at the optimal point. And then when we convert that into our required output these are the numbers which I have already presented before I started showing this MATLAB code in detail.