Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –173
Capacity Demand Component Reliability (Part 21)

We have solved two variable problems two random variable problems in the previous lecture but now we graduate to three variable problems and then to 4 variable problems in this lecture we stick with the same setup the cable intention that we have been looking at in various ways. So this is the problem with the third random variable defined.

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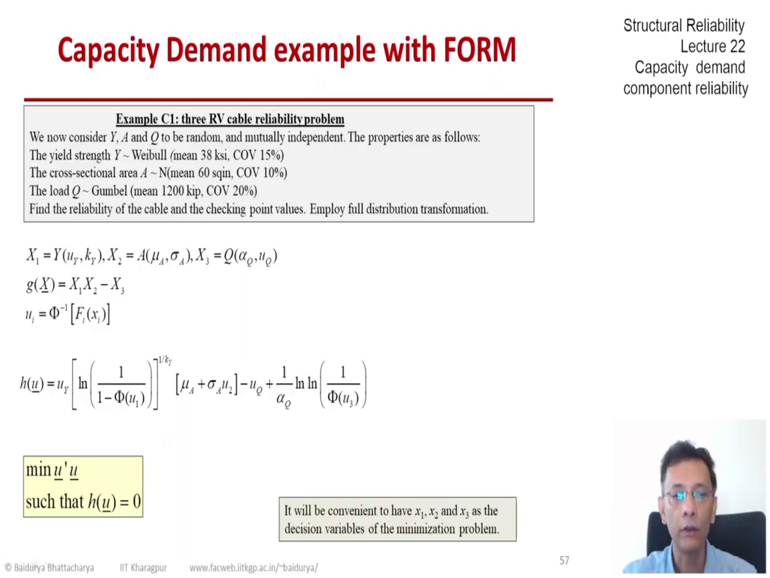

So, let us take a minute to read through the problem. So, we have three mutually independent random variables Y the yield strength A the cross-sectional area and Q the load they are respectively Y is viable a is normal and Q is gumball. Earlier in the last one or two examples we fixed Q at the mean value 1200 k now Q is a gumball random variable. We no longer wish to use the second moment transformation we lose our probabilistic information if the variables are not normal.

So, here we would like to employ the full distribution transformation. So, let us take that up. So, X 1 let us call X 1 Y or Y X 1 a X 2 and Q X 3 our limit state therefore becomes X 1 X 2 – X 3. So, it is a simple looking limit state now as required the distribution transformation is u is phi inverse of F of x. So, that is for each of the x's. So, i going from 1 to 3 basically you can interpret this as phi of u is F of x.

So that is a distribution wise equivalence. Now if we wanted to find out h express h in terms of use and then find the minimum distance to that h equals zero. So, this is how you would proceed let us do variable by variable. So, X 1 can be given in terms of u 1 in a rather complicated nonlinear fashion which is what you see this is the this is the result of equating the viable that two parameter viable with the normal.

And once you go through the algebra this is what you end up with the viable distribution looks like 1 minus exponential of negative X over u to the power of k. So, once you solve it this is the form it comes to the next one would be much simpler because area is normal. So, X 2 in terms of u 2 is a simple linear transformation and then when we come to the gumball we also have a rather complicated looking transformation and that is what you see on this screen the gumball form uses the double exponential form.

So, if you want to go through the algebra and work through step by step this is what you are going to find. So, in the first one and in this third one we see that the normal cdf is implicitly there in the new limit state function. Obviously this is a complicated limit state function and to find the minimum distance to this from u equals 0 is going to be complicated we could do that but there is not a single way of solving a problem.

So, what I want to show you in this one is in this example is why do not we stick to the basic variable space things look more manageable here more intuitive our limit state function is quite simple looking X 1 X 2 minus X 3 instead of something as complicated when we go to u space. So, why do not we see what happens if we stick to the X space and try to do the same thing. Obviously we cannot optimize we cannot minimize the distance X 1 square plus X 2 squared plus X 3 squared that would not be right.

We still have to do u 1 square plus u 2 squared plus u 3 squared we have to minimize that but we can operate from xbase. So, that is our whole power and let us sees how far we can go with that.

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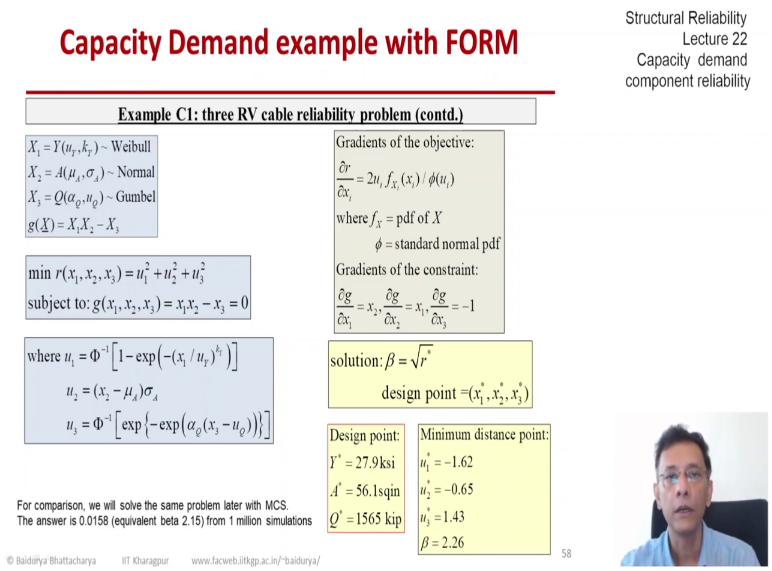

So, this is our problem statement we have g of X equals X 1 X 2 minus X 3 and now we want to minimize the function of X 1 X 2 X 3 which is our distance in the standard independent channel normal space u 1 square plus u 2 square plus u 3 square. Obviously there is a relation between the x's and the u's the constraint is also put in the basic variable space. So, we have a nice looking constraint and equality constraint X 1 X 2 minus X 3 is equal to 0.

And here are the function relations between the u's and the x's. So, as I was saying that this is the weibull to normal transformation this is the normal to normal transformation for u 2 and for u 3 this is the gumball to normal transformation. So, this is our problem set up and but we would need if we could provide it would be helpful we would need the gradients of the objective if we go for a gradient-based algorithm we would also need the gradient of the constraint if we can then that will be great the problem becomes more efficient to solve.

So, let us look at it how how to work this. So, the objective function r which is the distance to the

the distance to h in the u space. So, that u1 squared plus u 2 squared plus u 3 squared the first derivative of that the first partial the gradient of that in respect to the axis would be we can use the chain rule of differentiation. So, twice u and then del u del x. So, del u del X for that we will use the distribution to distribution equivalence.

So, that term that you see the density of X divided by the density of u that basically is the product of equating the the two cdfs. So, phi of u is cdf of x. So, we differentiate the left hand side with respect to X. So phi of u with respect to u first then partial u partial X and on the right hand side it is the partial of the cdf with respect to x. So, that gives me the density of x. So, that is how this twice u pdf of X divided by the pdf of u that form comes in the final solution.

And the gradient of the constraint in this particular case it is very simple the first one is X 2 the second one is X 1 and the third one is -1 you just differentiate g and that is what you get. Now our solution would be if we can find the minimum of r our beta would be the square root of that minimum. So, root of r star and the design point automatically since we are operating in X space would be X 1 star X 2 star X 3 star.

If you want to find u 1 star we have to use the map that you see on the left. So, u 1 is fine inverse of 1 minus exponential so on. So, that expression has to be used if we want to find the u 1 star the u 2 star and u 3 star 1 by 1. Here is the answer first and then I will walk you through the steps that I have used in obtaining these answers and that is basically employing a MATLAB program and it is a good way to understand how to set up such problems in a standard platform such as MATLAB.

So, the answer we get the design point is Y star is about 28 ksi the a star is about 56 square inch and the design value of the load is about 1565 kilo pounds. The corresponding minimum distance points in u space are as you see minus. So, 1.62 standard deviations below the mean for the first one about 60% standard deviation below the mean for the second one and about one and half standard deviations above the mean for the third one and that gives me a beta of about 2.26.

Now how accurate is this 2.26 beta of 2.26 we have mentioned that FORM is approximate. So, to compare we later have solved this problem using Monte Carlo simulations with a good number of trials and we get the value of P f about 0.0158 which gives an equivalent beta of about 2.15. So, it seems that forum is overestimating reliability or underestimating the P f and this is something we will remember when we take up a discussion of SORM later in this lecture.