

Structural Reliability
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Lecture –172
Capacity Demand Component Reliability (Part 20)

Welcome to this third lecture in this series on FORM, F-O-R-M the First Order Reliability Method. In the first one we discussed in detail the mathematical basis the algorithm and the limitations of FORM. In the second one which is the previous to this one we went through several problems in detail and then today we are going to look at two more and then in the remaining time we will take up the second order reliability method SORM.

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Recap: FORM steps

\underline{u} = independent standard normal space
 Map \underline{x} onto \underline{u}
 hence $g(\underline{x})$ onto $h(\underline{u})$

minimize $\|\underline{u}\|$
 subject to $h(\underline{u}) = 0$

Solution, \underline{u}^* = checking point
 $\beta = \|\underline{u}^*\|$ = reliability index

$P_f \approx \Phi(-\beta)$

More generally:
 $P_f \approx \Phi(-\beta \text{sgn}[h(\underline{0})])$

Key steps

- Map from \underline{x} to \underline{u}
- Find minimum distance from origin to $h(\underline{u}) = 0$

Points to note

- h not required in closed form - point wise OK
- Gradients of h not essential
- Many optimization schemes available
- Non-linear optimization - global optimum not guaranteed

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So, let us recap the essential features of FORM we start by mapping the basic variables onto the independent standard normal space. So, x to u and hence we get a new limit state equation $g(x)$ equals 0 to $h(u)$ equals 0. And our purpose is to minimize the distance to this limit state equation in the standard normal space in the independent standard normal space and that minimum distance point is the checking point or minimum distance point most likely point these are the various names and that distance is the reliability index.

So, that in an approximate sense which we are going to probe in detail a little later in this lecture the failure probability is ϕ of minus β or the reliability is ϕ of β . To be more correct we need to take the sign of the limit state at the origin the whether the origin is in the safe set or not and but typically it is. So, we ignore that signum function pictorially speaking this is what we do we map x onto u and the new limit state h equals 0 in terms of u .

And the PDF contours are nice concentric circles around the origin in u space and we find the minimum distance β to the minimum distance point u^* . To summarize the key steps and some of the pros and cons are we have to map x onto u and there are several possible maps we mentioned about five in earlier in this series the second moment transformation are the Hasofer-Lind transformation, the full distribution transformation, the Nataf transformation the Rockwood's Fisher transformation and the Rosenblatt.

In fact if you remember all the problems we solved in the previous lecture used the Hasofer-Lind transformation we are going to look at other transformations today. We and then the next step is the optimization part which is the minimizing the distance. Now the pros and cons as I mentioned that we do not need h in closed form if there is a method like a finite element program which gives a h point by point that would be we do not need to use a gradient based algorithm to optimize the minimum distance.

There are many optimization schemes available and the for example the evolutionary type algorithms do not use gradients but importantly this is a non-linear operation problem. So, there is no guarantee that the global optimum is obtained. So, we can always find a local optimum and only if it is a convex operation problem we can be assured that the local optimum is the global optimum. So, now let us start solve the problems which involves different maps.