Structural Reliability
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Lecture –171
Capacity Demand Component Reliability (Part 19)

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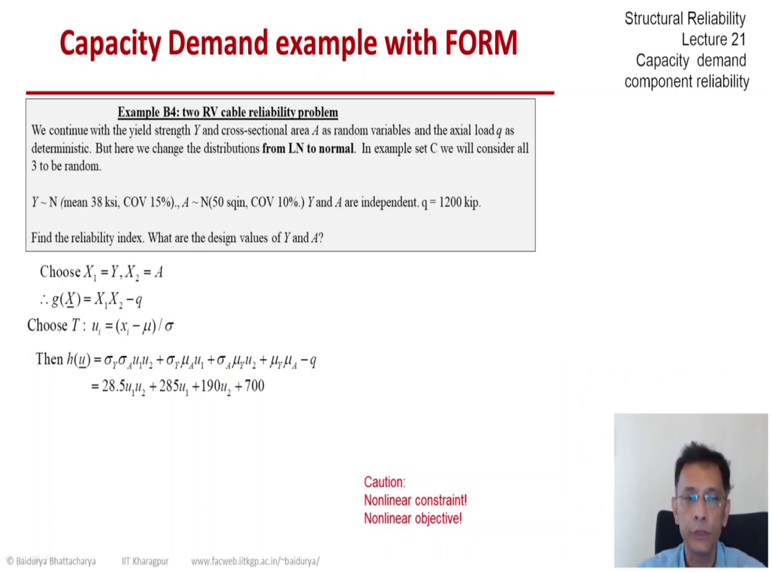

In the three problems that we have solved so, far with FORM the limit state always turned out to be a linear function it does not have to be so, always obviously. So, let us take a look and see what we would do if we encountered a non-linear limit state and B4 gives us an opportunity. So, it is the same problem in terms of the element definition it is a cable under tension the yield strength is random the cross-sectional area is random.

The load is deterministic because we want to stay confined to two random variable problems yet but the distributions of Y and A are not log normal as we did in the previous example but they are both normal. So, now immediately there is a problem we perceive is that we cannot take log of X of Y and log of A and come up with a distribution that is normal or any other well-behaved distribution.

First of all you cannot take logarithm of a normal random variable because it takes on negative values and secondly even if we truncated it at zero it still would not help us because that would not give us a unknown distribution or density function. So, we have to accept that we are going to deal with a non-linear limit state. So, let us just do a simple Haseful-Lind transformation of X 1 and X 2, X 1 being Y itself and X 2 being A.

So, with the second moment transformation our limit state in u-space becomes a non-linear function. So, the constants come from the mean and standard deviations of Y and A also there is one important difference here is we have not made any requirement that the failure probability or the reliability index be of a certain value we have just presented the problem given the properties of Y and A. And here all we want is to find what the reliability index is we just want to find it and then whether we want to move mean of A around to achieve a particular beta that we will take up later.

But for now let us just see how we can solve this nonlinear limit state problem. Obviously the way we did the previous three is just have the minimum distance to a straight line from the origin it is not going to work anymore. So, we have a non-linear constraint and a nonlinear objective function. The way to go about would be this gives us an opportunity to show the application of the gradient projection method that I presented in the previous lecture.

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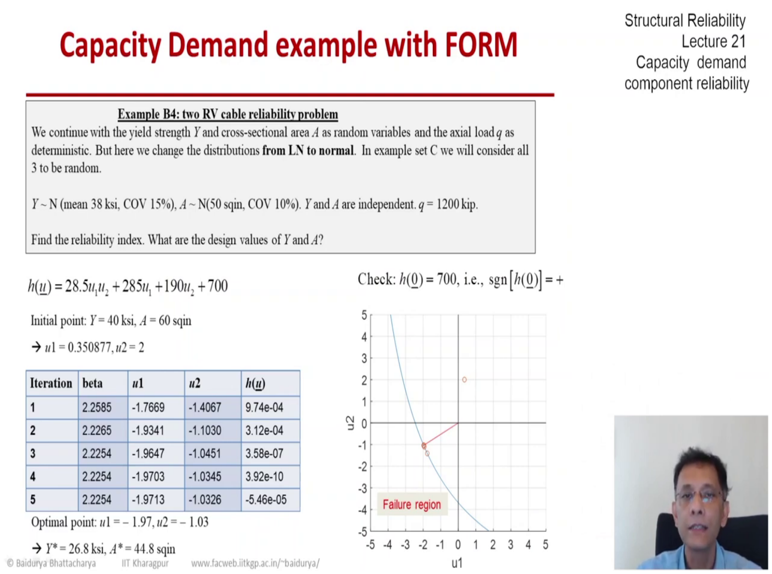

So, let us proceed step by step this is our limit state equation and this is a limited function and if we plot it on u1 u 2 space we see that it is a line a curved line and we are going to find the minimum distance to this blue line this is the failure region to the left of the blue line and just to make sure that that is the failure region we can see whether the origin is in the failed or the safe set. So, putting the u1 and u2 as 0 we see that the function is positive. So, the origin is definitely contained in the safe set.

So, we have that behind us and now we start the optimization process we could choose any initial point it is it is good to choose a point that would not take us to the origin because it might be very difficult to move away from there. So, let us say we choose Y as 40 and A as 60 in the space of physical or basic variables and that gives us a u1 and a u2 that you see on the screen and that is also marked with the red arrow on the first quadrant.

So, our choice has been such that we start from very much inside an interior point in the safe set but let us now see how the iteration proceeds. So, this is the steps that we are going to follow in the gradient projection algorithm. So, after that it is good to know that the very first iteration ends with you back on the limit state line the red arrow shows that and we have already come very close to the optimum u 1 is at minus 1.76 u2 is at minus 1.40 we are pretty much on the limit state surface limit straight line h is very close to zero 10 to the power minus 3 roughly and beta the distance at this stage is about 2.26.

So, let us proceed we move closer to the optimal point from minus 1.76, u 1 moves to minus 1.93. So, u1 moves a little further away u 2 moves from minus 1.4 to minus 1.1 the point is still very much on the limit state line it is about 3 10 to the minus 4 and beta comes down to 2.23 and then in the next iteration we have pretty much converged already u is -1.96 and u2 is -1.045 beta is almost unchanged 2.2 to 5 and h is even smaller.

So, we are again very much on the line and we are traveling down the line or up in this case the next iteration u1 moves a little further away u2 moves a little closer beta is practically unchanged and the functional value is very, very close to zero and now we have converged all tolerances have been met. So, the answer is beta is 2.225 and the u 1 and u 2 star values give us the physical variables at the design point as 26.8 ksi for yield and 44.8 square inch for area.

So obviously in this situation where the mean of the cross section area is 50 square inches we end up with reliability much lower than 3. So, now obviously if we got back to the mode of all the previous problems b 1, b 2 and b 3 and said that no this is not good enough beta of 2.22 is not going to work we have to have a beta of 3.

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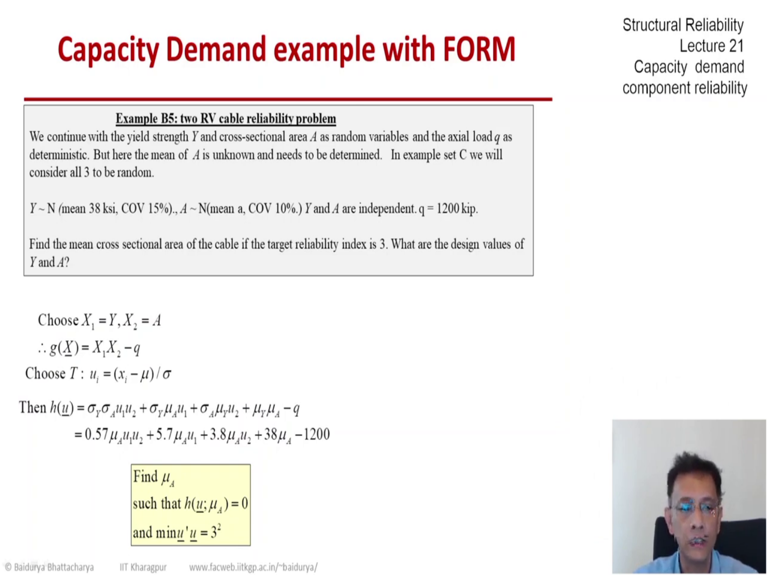

So, in that case we now have a nested optimization situation and that brings us to problem B5. And now the question is again we want beta of three and so, what mean area would achieve that. So, we are letting the mean to be selected which basically means from a practical viewpoint that we can choose a thicker or a thinner cable with the same material. So, we have to optimize beta as a function of area or mean area but even to find beta we have to have one more optimization. So, that is why I said nested optimization.

So, as before we choose X 1 as Y and X 2 as A and it gives us the limit state equation limited function in u space as you see we have the mean of a not defined. So, that is that is one of the unknowns and so, we have to find that mean of A. So, that it is a constraint optimization and the distance is minimized. So, this in the yellow block what you see is the problem statement.

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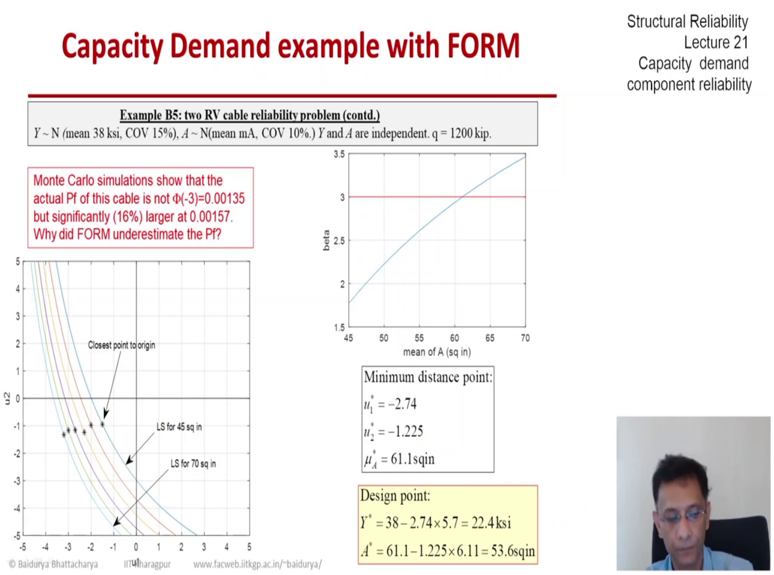

So, let us see what results we get what we have here is after putting in all the values of the mean and standard deviation of y and a and mu a is the unknown here we get a family of curves. So, all those lines that you see previously we just have 1 corresponding to the mean of a as 50. Now we have a family of mean of a starting from 45 square inch all the way to 70 square inch and all the stars that you see on these limit state lines all those marked as closest point to the origin.

So, each of those curves has one optimal point the point of maximum likelihood the checking point the distance would be the corresponding beta value. So, each of those gives rise to one beta value for the; corresponding mean area. So, if we put all of them together if the beta is extracted for each of those curves we get a relationship between beta and the mean of a in the previous problem in b4 we actually solved beta as about 2.22 when mean of a was 50 square inch.

But as I said here we let the mean of a varies between 45 and 70 and we see where it gets to better three first. So, it so, happens that if you see the graph on the right the beta of three is achieved when the mean of A is roughly about 60 one or 62 square inches. So, that would be the answer for the checking point values and the corresponding optimal mean area that gives beta of 3. If you are interested in the design point the design point for yield is 22.4 ksi which is something we have been observing quite a lot in that region.

And the design value for area is about 54 square inches. So, the mean has to be something like 61 and the design value a little lower than the mean would be 54. Now I would like to leave you with a thought and we are going to come back to this is when we do Monte Carlo simulations for the same problem we see that the actual P f of this cable is not phi of minus three phi of -3 gives very close to 0.001 but it is actually much larger.

So, form is underestimating the failure probability instead of 0.001 which is what form things the actual value is as I write here 16% larger. So, why is that the question is why is form underestimating the P f and we are going to tackle this in the next lecture where we take up more problems with FORM and then spend some time discussing SORM second order reliability method and following which we are going to take up Monte Carlo simulations.