

**Structural Reliability**  
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**Lecture –169**  
**Capacity Demand Component Reliability (Part 17)**

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**Capacity Demand example with FORM**

Structural Reliability  
 Lecture 21  
 Capacity demand  
 component reliability

**Example B2: two RV cable reliability problem**

We continue with the cable design problem, and change the distribution of both  $Y$  and  $Q$  from normal to lognormal: Yield strength,  $Y \sim \text{LN}$  (mean 38 ksi, COV 15%). Axial load,  $Q \sim \text{LN}$  (mean 1000kip, COV 10%).  $Y$  and  $Q$  are independent. Cable failure is defined as yield of the gross section.

Find the cross sectional area of the cable if the target reliability index is 3

What are the design values of  $Y$  and  $Q$ ?

Note: We have already solved this problem analytically in Example A2. Here we take the longer route in order to learn the steps of FORM.

Choose  $X_1 = \ln Y, X_2 = \ln Q$

$\therefore g(\underline{X}) = X_1 - X_2 + \ln a$

Choose  $T: u_i = (x_i - \mu) / \sigma$

Then  $h(\underline{u}) = \sigma_{mY}u_1 - \sigma_{mQ}u_2 + \mu_{mY} - \mu_{mQ} + \ln a$   
 $= .149u_1 - 0.0998u_2 + 3.63 - 6.90 + \ln a$

Minimum distance from origin:

$$\beta = \frac{\ln a - 3.27}{\sqrt{0.149^2 + 0.0998^2}} = 3 \text{ (required)}$$

Solving,  $a = \exp(3.81) = 45 \text{ sqin}$

Minimum distance point:

$$u_1^* = -0.149 \times 3^2 / 0.54 = -2.5$$

$$u_2^* = +0.0998 \times 3^2 / 0.54 = +1.7$$

Design point:

$$x_1^* = 3.63 + 0.149u_1^* = 3.26$$

$$x_2^* = 6.9 + 0.0998u_2^* = 7.07$$

$$Y^* = \exp(x_1^*) = 26 \text{ ksi}$$

$$Q^* = \exp(x_2^*) = 1170 \text{ kip}$$



The next problem we solved with FORM is one that we have looked at before which was called example B2 earlier this week but we are now going to solve it with flrm and go through the steps. How it is different from just the previous example is instead of normal distribution for  $Y$  and  $Q$  we have log normal distribution for both of them. And we will see what difference it makes to the results.

So, let us just take a few seconds to read the problem and then we will start the solution. So, it makes sense as we had discussed when we solved this problem before is we take log of  $Y$  and log of  $Q$  and express them as  $X_1$  and  $X_2$ . So, our limit state function in the space of basic variables is  $X_1$  minus  $X_2$  plus log of  $a$ . So, it is a very simple linear function and we should go for a linear map. So,  $u$  is  $x$  minus  $\mu$  over  $\sigma$  actually this is a normal to normal transformation just like it was in the previous example B1.

So, we are not losing any information probabilistically. So, with this we get a similar limit state as we did in the previous problem it is a linear function involving  $u_1$  and  $u_2$  and as you see I have put the numerical values you need to go through the steps to get the equivalent mean and standard deviation for the corresponding normal starting from the log normal. So, I have already done those and what you see on the screen is the final result of the limit state equation in  $u$ -space in terms of all the given constants.

The one unknown constant is  $\log$  of  $a$ . So, part of the exercise is to find out what the value of  $a$  is if the reliability index has to be 3. So, it is the same problem as before we have to find the distance of a straight line from the origin and we have we did the same thing in the previous problem here it is given in terms of  $\log$  of  $a$  and if  $\beta$  has to equal 3 then that gives me a solution of  $a$  about a about 45 square inch which is the only feasible solution.

But interestingly our previous answer when both were normal random variables was 50 square inch. So, now we have 45 square inch and this is just a reminder that the answer to any reliability analysis not only depends on the mechanics but depends on the underlying assumptions in the probabilistic aspect of the problem as well as the method taken to solve the problem. So, this is the minimum distance and now we can find the coordinates of the minimum distance point and that is respectively minus 2.5 and plus 1.7.

So, the similar thing as we saw before is that the most likely point for the strength variable is lower than the mean it is near the left tail it is 2.5 standard deviations below the mean and for the load it is almost two standard deviations above the mean. So, it is 1.7 standard deviations above the mean and in comparison the numbers were a little different previously I think it was something like 2.7 for the strength and about just one for the load.

So, again these results depend on the distribution and the map. So, the design point is 3.26, 7.07 in  $x$  space and if we now exponentiate them to get back  $Y$  and  $Q$  the values are 26 ksi and 1170.

So, these numbers are different from the previous example and as I am saying it is because of the distributions of Y and Q are different.