

Structural Reliability
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Lecture –168
Capacity Demand Component Reliability (Part 16)

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Capacity Demand example with FORM

Structural Reliability
 Lecture 21
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 component reliability

Example B1: two RV cable reliability problem

We go back to designing the cable in a suspension bridge made of A36 steel. Yield strength, $Y \sim N$ (mean 38 ksi, COV 15%). Axial load, $Q \sim N$ (mean 1000kip, COV 10%). Y and Q are independent. Eventually, we will extend this to a three-RV problem by considering the area to be random as well.

Find the cross sectional area of the cable if the target reliability index is 3

What are the design values of Y and Q ?

Note: We have already solved this problem analytically in Example A1. Here we take the longer route in order to learn the steps of FORM.

Choose $M = g(Y, Q) = g(\underline{X}) = aY - Q$

Choose $T: u_i = (x_i - \mu_i) / \sigma$

Then $h(\underline{u}) = a\sigma_1 u_1 - \sigma_2 u_2 + a\mu_1 - \mu_2$

Need to find minimum distance to
 $h(\underline{u}) = 0$
 from the origin

Consider the straight line:
 $ax + by + c = 0$

Its distance from the origin is:
 $d = \frac{c}{\sqrt{a^2 + b^2}}$

The point on the line closest to the origin is:
 $x^* = -\frac{ca}{a^2 + b^2} = -\frac{ad^2}{c}$
 $y^* = -\frac{cb}{a^2 + b^2} = -\frac{bd^2}{c}$

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The first problem we solve with form we have named it example B1. It is a problem we have actually already solved before earlier in this week this involves that cable under tension. And now there are two random variables one is the yield strength the other is the actual load. We have solved this but just let us take a few seconds to recap and then we will solve this problem with f y. So, it is it is a known problem but now we will use a different method to solve it.

There is a new question here is what the design values of Y are and Q and we will come to that at the end of that solution. So, we have to choose a safety margin and the form that we choose is quite intuitively that it is capacity minus demand. So, it is a times Y - Q a is non-random at this point and we actually have to find the value of a the transformation we choose again we choose the simplest one that has a line transformation the second moment transformation. So, all we do is u is x - mu over sigma.

So, there are 2 x's. So, we have 2 u's and it is a two dimension problem. So, you can actually plot it if we want on a paper. So, this gives us after employing this transformation that the line the h the new limit state function is a linear function in u 1 and u 2 a sigma Y sigma Q mu Y mu Q these are all non-random constants. So, we basically have to find the minimum distance to the straight line from the origin.

And now let us just recap some of the lessons from coordinate geometry long back. So, if we have a straight line $ax + by + c = 0$ in two variables x and y then we can look up or we should be able to remember that the distance of the straight line from the origin is $c / \sqrt{a^2 + b^2}$. So, that that is something we I am sure all of us did a long time back and then the point which is something else we also need the point on this line closest to the origin is x^* and y^* and you can either give them in terms of a, b and c or a, d and c or b, d and c respectively.

So, we have to remember these results and we have to now go back to this line h equals 0 and find out both the minimum distance and the coordinates of the points. So, d is $c / \sqrt{a^2 + b^2}$ and x^* is $-a d / (a^2 + b^2)$ and y^* is $-b d / (a^2 + b^2)$.

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Then $h(u) = a\sigma_Y u_1 - \sigma_Q u_2 + a\mu_Y - \mu_Q$

Minimum distance from origin:

$$\beta = \frac{a\mu_Y - \mu_Q}{\sqrt{a^2\sigma_Y^2 + \sigma_Q^2}}$$

Solving, $a = 50 \text{sqin}$

Minimum distance point:

$$u_1^* = -\frac{a\sigma_Y(a\mu_Y - \mu_Q)}{a^2\sigma_Y^2 + \sigma_Q^2} = -2.8$$

$$u_2^* = -\frac{-\sigma_Q(a\mu_Y - \mu_Q)}{a^2\sigma_Y^2 + \sigma_Q^2} = +1.0$$

Design point:

$$Y^* = x_1^* = \mu_Y + \sigma_Y u_1^* = 22 \text{ ksi}$$

$$Q^* = x_2^* = \mu_Q + \sigma_Q u_2^* = 1100 \text{ kip}$$

Prescribing the design equation as:

$$aI^* > Q^*$$

to designer would ensure $a = 50 \text{sqin}$ and $\beta = 3.0$ without the need for first principles based reliability analysis!

Finally, Y^* and Q^* can instead be presented to designer in terms of factored characteristic values



So, with these formulas in mind now let us put these back on the problem at hand and. So, the minimum distance from the origin beta is given in terms of the constants of the problem the cross-sectional area the means of the random variables and the variances of the random variables it is also required that beta is equal to 3. So, if you if you solve it the answer comes to a as 50 square inch now with that answer we can plug it back and get u 1 star the coordinate of the minimum distance point is negative 2.8 and the coordinate of u 2 star is plus 1.

Now this itself is actually very helpful it tells us that the most likely value the point with the highest probability on the limit state line is almost three standard deviations below the mean for the strength variable which is u1 because u1 is a map from the eel strength and u 2 is a map of the load the most likely value of the load is one standard deviation above the mean. So, this is actually a very useful by product if I may use the word of form not only does it give you the reliability index but it tells you which point is most likely to happen near about failure.

So, that we; can use it in kind of a design sense which I am going to discuss just in a few seconds. So, this is the design point how do we do that we back x 1 star from u1 star and x 2 star from u 2 star. So, the most likely value of Y the yield strength is roughly 22 ksi the mean being 38 ksi and the most likely value of the load is about 1100 kilopounds the mean being 1000 kilopounds.

So, this is actually quite useful information because this point Y star and Q star they are on the limit straight line in the basic variable space. So, now here is the idea about how to use this design point or checking point in design is that we can now say that being safe is you know g greater than zero. So, if we can say that a y star would be greater than q star and present the designer this equation to a designer to ensure then automatically this gives me a value of 50 square inch for the cross-sectional area all I have to do is that tell the designer that you know these are the design point values.

So, this way without even doing reliability analysis or knowing the steps this design equation

would ensure that a beta of 3.0 is observed. So, this is an indirect way of achieving a reliable index in design when the designer does not necessarily need to know the background reliability analysis process but a design equation if formulated properly can ensure that. So, that is and then later on we will see that we do not have to present these design values per se.

But we can present them in terms of nominal values which the designer is more comfortable with and present some factors to multiply these nominal values to get Y star and Q star to satisfy the purposes of reliability.