

Structural Reliability
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Lecture –164
Capacity Demand Component Reliability (Part 12)

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FORM steps and benefits

\underline{u} = independent standard normal space
 Map \underline{x} onto \underline{u}
 hence $g(\underline{x})$ onto $h(\underline{u})$

minimize $\|\underline{u}\|$
 subject to $h(\underline{u}) = 0$

Solution, \underline{u}^* = checking point
 $\beta = \|\underline{u}^*\|$ = reliability index

$P_f \approx \Phi(-\beta)$

More generally:
 $P_f = \Phi(-\beta \text{sgn}[h(\underline{0})])$

Key steps

- Map from \underline{x} to \underline{u}
- Evaluate $h(\underline{u})$ repeatedly
- Find minimum distance from origin to $h(\underline{u}) = 0$

Benefits

- h not required in closed form - point wise OK
- Gradients of h not essential
- Many optimization schemes available
- Gives "checking" point \underline{x}^*
- Very few calls to expensive FEM runs

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We have answered both questions that we started with the first one was where to map from the basic variables and the answer is on to the space of independent standard normal variables. And how to map there is no single answer there are various possible maps some of them require more information and most of them distort the limit state and affect the probability of failure. For most cases the deviation or the error is not too much.

So, the choice of the map is often a matter of convenience and obviously available information now. So, so let us recap what we have discussed. So, we map X onto u and g onto h we minimize the distance from the origin to h equals 0 and then that solution is known as the checking point the maximum likelihood point also called the design point and we will come to that later why it is called the design point.

And that minimum distance is the reliability index. So, that the failure probability is approximately $\Phi(-\beta)$ where Φ is the normal CDF. Why it is approximate we discussed partly because the approximations introduced by the map and secondly the linearization that we do in the standard normal space the linearization of the limit state function and obviously we discussed whether the origin is in the failure domain or not that should be considered strictly speaking now just to put all of that together.

So, the key step is a map we need to evaluate the limit state repeatedly. So, some structural analysis some finite element call may be required and we need to do this until we are able to find the solution. There are many benefits of form apart from it being a very elegant analytical method we do not need h in closed form if we have any method which gives a point-wise answer that is good enough and that is why I mentioned FORM.

We do not have to have gradients of h but it definitely helps it speeds up the computation it either can be obtained analytically or if not possible then we obtain the gradients numerically. There are many optimization schemes available I mentioned and then we have to choose one we are going to talk more about that point then the form analysis in the end if you map back the u^* the checking point back to the basic variable space.

So, that gives us X^* in the basic variable the physical variable space that can be very useful in design we will talk about that later and one of the benefits is that unlike a Monte Carlo simulation based technique typically relatively few calls are made to the structural analysis program. So, these are many benefits of this form method now we are going to discuss how to minimize the distance from the origin to the limit state equation on limit state surface and we are going to discuss the gradient based method. So, let us start that.