Structural Reliability
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Lecture –163
Capacity Demand Component Reliability (Part 11)

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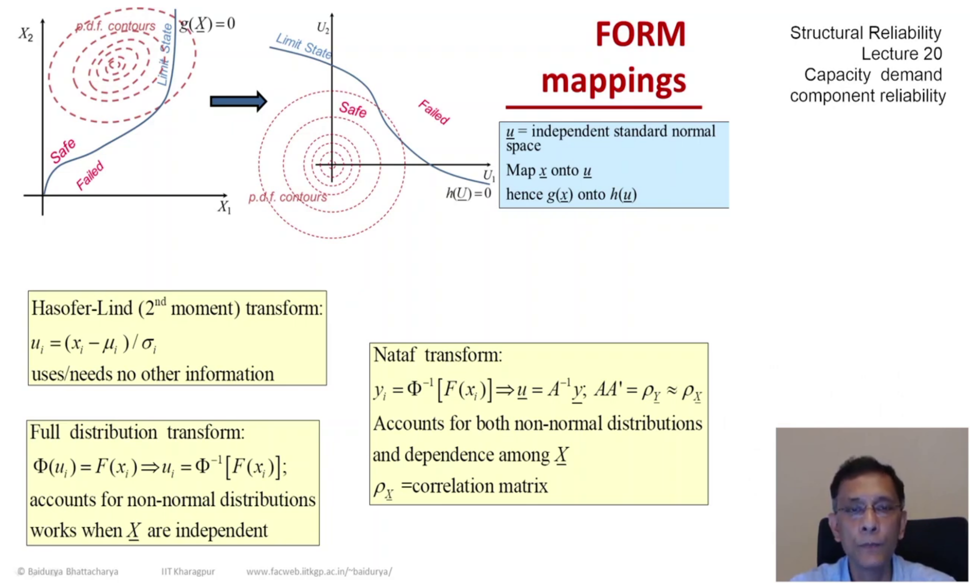

So, we started with two questions in this lecture the first was where to map from X to Y and what sort of map we have answered the first question is that we are going to map onto the independent standard normal space. So that is what you see on the top part of your screen is that we have the basic variable space on the left and there is a limit state there are the PDF contours and then for some map T we get into the independent standard normal space there is a new limit state equation and new PDF contours which happen to be the contours of the independent standard normal concentric circles around the origin.

So, the second question now is what sort of map turns out there are several possibilities and they are widely used. So, let us look at about five of them the simplest one is a linear map from X to u. So, this is called the second moment transform because the first and the second moments the mean and standard deviation are being used it is also called Hasofer Lind transom because those are the two authors who propose this first and what you see on the second line in that box is the transformation from X to u.

So, in effect what we do is we subtract the mean of X and divide it by the standard deviation of X and we get the u. So, we do it for every X and as you see this map uses or needs no other information which means is that if the X's were dependent there's no way of imposing or accounting for that dependence the excess can be very non-normal but this linear transformation has no way of capturing that clearly I am sure you recognize this transformation that we have used many times when we try to find the normal probability.

So, any arbitrary normal random variable X if we convert it to the standard normal this is exactly what we do and that helps us compute the CDF of X. So, here this map would be exact if X was a normal random variable and all the X's were independent. So, exact in the sense of preserving the probability information and preserving the nature of the limit state but this is a very simple transformation very popular and in many cases quite adequate.

So we are going to use this in the examples that we will take up in the next lecture. The next map is the full distribution transform and this acknowledges the fact that the excess could be non-normal in nature. So, what is lost in the Hasofer-Lind transformation is preserved here. So, here it is point by point full distribution transformation. So, every X is related to the corresponding u through equivalence in the CDF. So, u is phi inverse of F of X for each i or the reverse map would be X would be f inverse of phi of u.

So, this works great preserves the complete probabilistic information but if the excess had any dependence this member by member transformation obviously cannot capture that dependence. So, this works great when the when the X's are independent obviously what we would need we would need to know the functional form of capital F and we should be able to invert it inexpensively as long as we can do that this method should work great it does impose an additional cost because the limit state h in the new space u might get a little more complicated looking because of all this nonlinear transformation involving phi and f.

The next one is Nataf transformation and this actually acknowledges the fact that there could be dependence in the access and in a rudimentary manner these attempts to address that point. So, what we do here is the there is an intermediate step. So, from X we go to Y and from then Y we go to u. So, to go from X to Y it is basically the full distribution transformation. So, Y is phi inverse of f of X that is fine but then we do something clever for going from Y to u and there we create a linear combination.

So, the u's are a linear combination of the Y's or conversely the Y's are a linear combination of the use and what is the nature of that linear combination what we do is we create the Cholesky factor of the correlation matrix of the Y's and impose that so, that that dependence on the Y's are realized what do you mean by that if you invert the equation in that second line of the net f transform box which is u equals a inverse Y it is basically Y is a of u.

So, Y is a linear combination of the u's, u themselves are independent standard normal but because of the presence of the matrix the y's become dependent because they are combining the same use. So, the Y vector has a correlation matrix rho y and now if I transform y's point by point to X or X to Y then the X's naturally have a dependence imposed upon them. There is one subtle point there is that the dependence imposed upon the X’s would be different from that between the Y's in terms of the correlation matrix because all we are doing here is imposing the correlation structure.

But for most cases this difference between the rho of Y that we start with and the row of X that we get is not much if you want to be very strict about this then there could be an iterative process in which we find out what changes in rho Y would give me the exact desired rho X that we know. So, that way we would be more true to the correlation structure of the X’s. Now obviously this sort of dependence among the X's is incomplete because correlation coefficient only gives me a linear dependence information.

But that is not a big problem from a practical point of view because in most cases we do not have information beyond the linear sort. So, all we have would be the CDFs the marginal CDF of the X’s and the correlation covariance matrix and then this Netaf transformation would be very appropriate for mapping from X to u.

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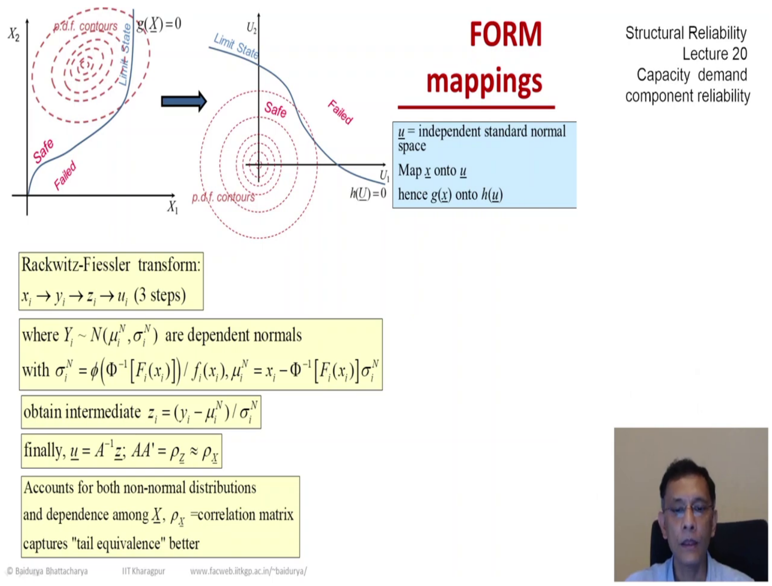

If we insert an additional step in the Netaf transformation there is a way of capturing the tail equivalence the tail of interest between the X space and the u space and that is known as the Rackwitz-Fiessler transfer. So, let us look at the steps. So, we start with X then we get a Y then from Y we get a Z or a Z and then finally the u. So, how are the Y’s obtained the Y's are the new intermediate variable which tries to create equivalent normal mean and equivalent normal standard deviation for each of the X's.

So, you see the equations there and. So, you can see point by point we are fitting that. So, for each value of X there is a particular equivalent sigma and a particular equivalent mu and that would give me the Y and then we can standardize that. So, the new standardized Y we are calling Z. So, that is Y minus the mean divided by the standard deviation. So, from X we go to the equivalent normal Y from there we get to the equivalent standard normal Z.

Now so, far we have not talked about dependence but now we are going to bring back what we did for the Netaf transformation. If we know what the correlation structure is then we would take the factor A for the correlation matrix and then we would linearly combine them to get the use R inversely we would express the Z's in terms of a linear combination of use and thereby impose the dependence which can carry all the way to X's obviously because of the series of nonlinear transformations rho of Z is not going to be same as rho of X.

But we can adjust them to capture the true correlation structure now whether we are talking about Rackwitz-Fiessler or Netaf it is not necessary that rho has to be there we could very well apply native and Rackwitz-Fiessler transformations if the X’s were independent but they do give us an opportunity of imposing a partial measure of dependence between them between the X's. So, just to summarize this Rackwitz-Fiessler transformation can capture the tail equivalents better although it introduces a certain amount of computational burden an extra step in the process.

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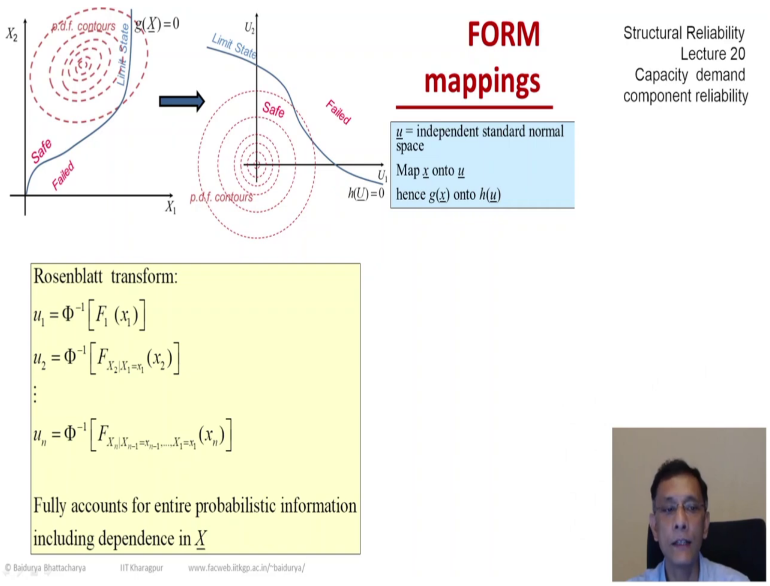

The final map that we look at is complete in every sense it is the Rosenblatt transform. The problem is that we typically do not have information to this extent to affect this sort of transformation. So, if you see how this is executed we do it step by step. So, the very first equation transforms x1 to u1 no problem there but then we have to use the dependence. So, we have to use the conditional CDF of x 2 given that x 1 has taken that particular value. So, clearly because of f of x 2 given x 1 depends on what x 1 is so, u 1 and u 2 naturally have a dependence imposed on them.

And then we proceed so, x 3 would be. So, x 3 given x 1 and x 2 and then that would give me u 3 and the transformation in each case is the full distribution type transformation that we have already seen. So, phi of u is F of X except the the f of X is now conditional CDF except for the very first one x 1 and we do it all the way until we reach the last one which is f of x n given all the previous values if you know that conditional CDF then we can invert that through the normal CDF and obtain the last member of the u vector.

So, this uses the full probabilistic information including all dependent structure among the excess but it's more ideal in nature because it is practically very, very difficult to obtain the joint CDF of order n among the X's in a functional form.