Structural Reliability
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Lecture –162
Capacity Demand Component Reliability (Part 10)

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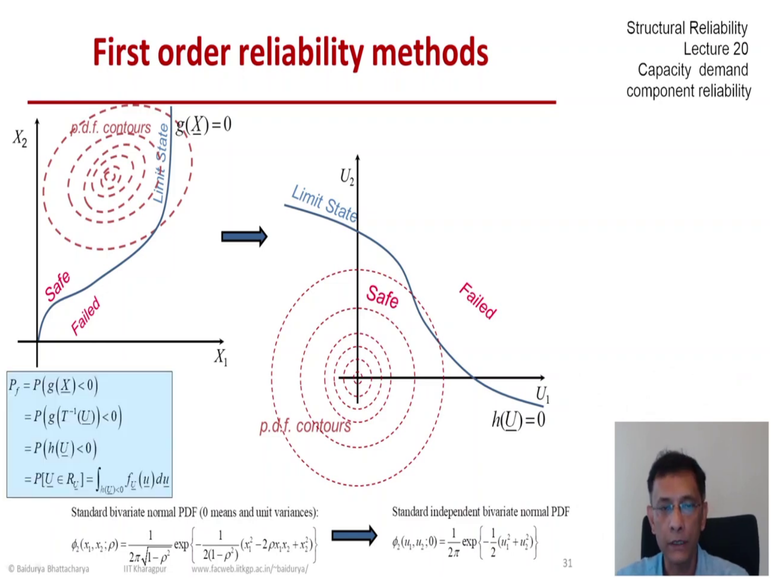

So this is what the map T does to the limit state and to also the probability density function in the new space. So, let us take a minute to study this new plot. So, on the left we have the basic variable space X and the limit state equation g of X equal to 0 and the probability density function of the axis what you see are the contours. So, they are basically cross sections we can imagine that the third axis coming out of the plane of your screen is the density axis the pdf axis.

So, this map T brings everything to this new y space. So, as I said two things happen one is the density function obviously moves and may take on a different shape depending on the nature of the map and also the limit state looks very different or could look very different it is curvature may change its location is most certainly to change and obviously we should go for a new symbol for the function. So, let us say g X maps to h of Y.

So, from now on we are going to see what to do with h of Y. So, that that is what our purpose is. So, just let us recap. So, we want to find the probability content of this region R y. Now that our limit state has been mapped to Y and we can find by some means this probability which we are hoping would be a much simpler and much more consistent exercise. So, then there is a very key question here and this was one of the breakthroughs in early structural reliability is what if this Y the space Y is the space of independent standard normal variables.

And let us give a special symbol for that which would be u. So, what if and this is the answer to one of the two questions that I raised in the previous slide where do we map what if we map into the space of independent standard normal variables. Something very interesting happens in that case let us see what the map looks like first. So, now u is our space of independent standard normal variables and from X we have mapped to u the limit state is now h of u that is all fine.

But what is very interesting is that the pdf contours in this u space are concentric circles centered on the origin. So, now that is actually very significant and we are going to take advantage of that soon in the next few slides. So, this is what happens to the probability of failure now we are interested to find the probability content in the region defined by R u the failed region that you clearly see on the right side of your screen and that is now a normal a joint normal probability.

And why are the pdf contours all concentric circles that is actually quite easy to see we have seen the joint normal density and the standard bivariate normal because we are talking about 2 random variables here. So, let us look at the bivariate as this example. So, this is the standard bivariate normal pdf that you see at the bottom of your screen x 1 and x 2 are two standard normal random variables with the correlation coefficient rho.

So, their means are each zero their variances are each one and you see that the expression in the exponential is like an ellipse. So, when rho becomes 0 then we have the standard independent bivariate normal and now the function in the exponential are just circular in nature. So, they at different heights they just give rise to circular contours and obviously are centered on zero.

And that as I said is going to be very significant when we compute the failure probability that we have been talking about.

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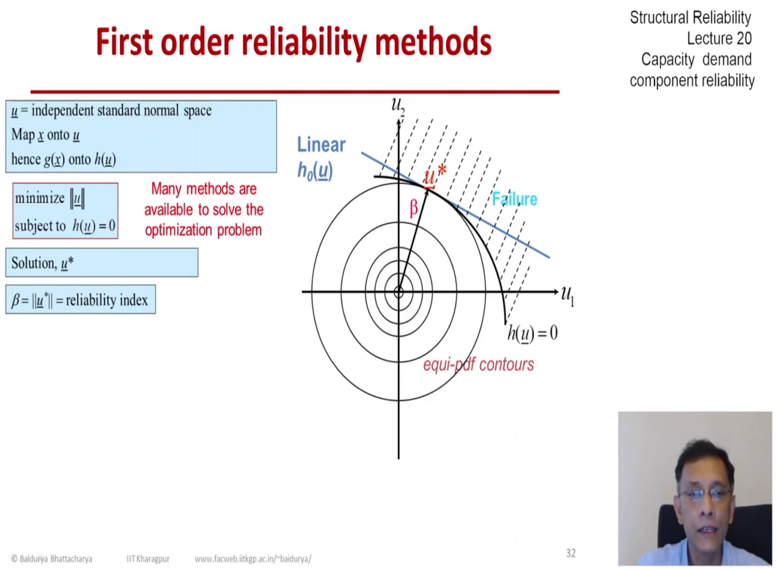

From this point forward we are going to stay confined to this independent standard normal space. So let us take stock of how far we have come we have mapped the basic variables X onto this independent standard normal space u and hence we have mapped the limit state function g of X onto another function h of u. And pictorially this is what looks like in two dimensions the equi-pdf contours are concentric circles around the origin and the failure region is clearly marked with dashed lines.

So, in form the objective is the problem statement is to minimize the norm of u subject to h equals 0 in other words we have to find the minimum distance point from the origin to the line h equals 0. And obviously this is an optimization problem and there are many methods available to solve such an optimization problem it's a constraint optimization problem what we are going to do later in this lecture and spend a good amount of time discussing this we will take a gradient based algorithms.

They are more classical in nature than more modern evolutionary algorithm based approaches for example but these are not only very elegant but they also give some insights into the problem which otherwise we would not get. So, we will look at gradient based methods actually these methods were developed in a time when Monte Carlo simulations were really not available to solve structural damage problems.

So these approximate geometry based first order liability methods were not only very useful but they are also very elegant and mathematically very sound. So, whichever method we use to solve this operation problem let us say the answer is u star. So, u star is a point on the limit state equation h equals 0 which is by definition closest to the origin. So, it has the minimum distance from the origin and that is what I have noted on the on the plot.

So, this minimum distance is called the reliability index and it is closely related to the failure probability or reliability that we are going to see next. To do that we must also actually mention that this beta corresponds to a failure probability not of the h equals 0 limits said that you see or the region defined by h equals zero but rather a hyperplane a line in this case of h 0 of u which is linearized which linearizes h of u at that minimum distance point.

So, there is an approximation and that linearization is actually the name that that lends the name first order in the first order reliability method. So, this beta is the reliability index and it is the minimum distance now why it is the failure probability or why is it the reliability.

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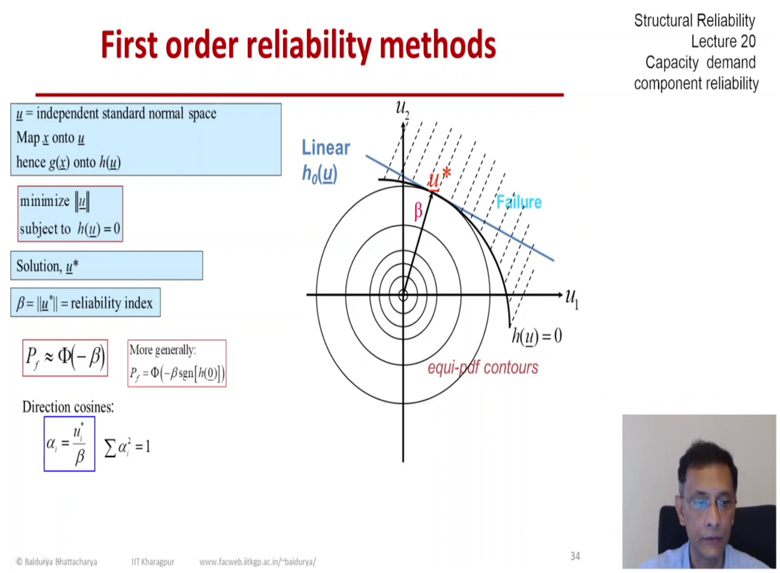

For that we have to make use of or invoke one of the fundamental properties of this independent standard normal space and that is something I alluded to a couple of slides ago is that the rotational symmetry of the independent standard normal space. So, let me see if I could just go back and forth with this what we are essentially doing here is we are turning the shaded region around the origin to now make that line beta that minimum search distance parallel to one of the axis say u 1.

So, it was like these located at the point u star and suppose we turn everything and because of the rotational symmetry the probability content does not change. So, the same probability in this shaded region is identical to the probability in this shaded region. So, that is a very useful and a very helpful if it is very simplifying property of this space. So, once more this is where we were this was my minimum distance point and the probability content of that shaded region is identical to the probability content of this shaded region or the probability content behind the linearized blue line is the same as that in this linearized blue line.

So, now obviously we can see clearly the problem is defined as the probability that u is u 1 is greater than beta that region is just a single one-dimensional normal cdf that we need to evaluate and that is why because of this rotational symmetry of the standard normal space the failure probability simply becomes the evaluation of a one-dimensional normal probability provided we can find that minimum distance point and the minimum distance.

Obviously this is an approximation as I already said it is the probability corresponding to that linearized that blue line that we had before again going back the it is h 0 that gives the modified failure region and that has an exact failure probability given by beta. So to put everything together our solution is u star our reliability index is beta and our failure probability is phi of minus beta because we are looking at a region away from the origin greater than beta.

So, it is phi of minus beta that is the failure probability and the approximately equal sign is due to the fact that we have linearized a nonlinear function h. So, if h was equal to h naught then that failure probability would be exact but it's not in general. So, that is where one of the approximations come from. It is also quite helpful to define the direction cosines which you see on the screen.

And they are also known as the sensitivity of each variable u, u i basically what that tells you is what is the change in reliability index or reliability for an incremental change in that particular u i. So, alpha i measures that larger is alpha the greater is the contribution of that random variable to the reliability you can map it back to X and find out which are the most important random variables in the physical space of basic variables contributing to failure.

Now it may have struck you that this problem setup does not differentiate whether the failure region is on the far side of h equals 0 or the near side in other words is the origin contained in the failure region or not. Obviously the origin being part of the failure region is it signifies very low reliability something less than even half which is not something we typically encounter in structural reliability but if you have to be completely accurate then we have to also evaluate that fact whether the the origin is part of the failure region or not.

So, if you like for the purpose of completeness we should define the failure probability as phi of minus beta times the sign of h at the origin. So, if the sign is positive then it is what we have been talking about if the sign happens to be negative which means origin is in failure then the failure probability becomes phi of beta and if beta is a large enough number then failure probability is going to be very high. In any case it is obviously going to be greater than half because beta is a distance it can never be negative.