

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –161
Capacity Demand Component Reliability (Part 09)

We ended the previous lecture with the idea that very few limit state probabilities can be solved in closed form. So, there was a need for approximate solutions and today we take up a very elegant method of an approximate solution which has come to be known as first order reliability methods. So, let us let us set the stage for that.

(Refer Slide Time: 01:03)

Recap: Limit state and failure

$Rel(t, \Omega; \Gamma, \Theta)$ = Probability that an item occupying a logical or physical domain Ω will perform its required function(s) Γ under given conditions Θ for a specified time interval $(0, t)$

The reference to the spatial dimension may be suppressed, and one defines failure as:

Failure = $\{M(\tau) \in \bar{\Gamma}_{sp}\}$, for any $\tau \in (0, t]$

In capacity demand formulation,
 $M(\tau) = f[C(\tau), D(\tau)]$, in general
 $= C(\tau) - D(\tau)$, when separable

Consider one failure mode at one critical location \underline{x} :
 Failure = $\{M(\tau, \underline{x}) \in \bar{\Gamma}_{sp}\}$, for any $\tau \in (0, t]$, for given $\underline{x} \in \Omega$

A "component" is an item of reliability that describes one failure mode at one critical location.


$C = C(\underline{X}_c; \tau), D = D(\underline{X}_D; \tau)$,
 It is not necessary that $\underline{X}_c \cap \underline{X}_D = \emptyset$
 Basic variables: $\underline{X} = \underline{X}_c \cup \underline{X}_D$
 $M(\tau) = g(\underline{X}; \tau)$

Time invariant basic variables

Component limit state equation
 $g(\underline{X}) = 0$
 such that $g(\underline{X}) < 0 \Rightarrow$ failed
 $g(\underline{X}) \geq 0 \Rightarrow$ safe

Probability of failure:
 $P[\underline{X} \in \bar{\Gamma}_{sp}] = P\{g(\underline{X}) < 0\}$

Structural Reliability
 Lecture 20
 Capacity demand
 component reliability



We are discussing in this part of the course a component reliability component reliability which is based on physics or failure. So, we have one failure element which is tagged by the location or the identity X and we therefore suppress the reference to X and we look at the safety margin m as a function of time and in general this safety margin is a function of capacity type and demand type random variables and those in turn C and D in turn are composed of other basic variables X sub C and X sub D .

So, together we call them the set of basic variables x . So, we could express the safety margin m

in terms of a function g of the basic variables now we are restricting ourselves to time invariant basic variables for now and. So, the problem simplifies to a limit state equation $g(X) = 0$ such that $g(X) < 0$ implies failure and $g(X) > 0$ implies a safe situation and what we are interested in is the probability that the basic variable is outside the safe set. So, the probability content of the basic variable space which come under the unsafe set.

(Refer Slide Time: 03:02)

First order reliability methods

Consider the failure region:

$$P[X \in \bar{R}_{\text{safe}}] = P(g(X) < 0) = \int_{g(X) < 0} f_X(x) dx$$

Let X have a one-one correspondence with Y
 $Y = T(X)$ and $X = T^{-1}(Y)$

Thus, the region R_x maps to R_y
then, $P[X \in R_x] = P[Y \in R_y]$

Under the map T :

$$P(g(X) < 0) = P(g(T^{-1}(Y)) < 0) = P(h(Y) < 0)$$

$$= P[Y \in R_y] = \int_{h(Y) < 0} f_Y(y) dy$$

Structural Reliability
Lecture 20
Capacity demand
component reliability

© Baidurya Bhattacharya IIT Kharagpur www.facweb.iitkgp.ac.in/~baidurya/
28

Now in two dimension this would be the representation of the problem the safe set versus the failed set and this is just copying the equation from the previous slide this is what we are interested in general X is an n -dimensional vector. Now obviously we have discussed this g does not have to be a linear function of the axis. So, we might as well bring in a clearly non-linear looking function and let us say this is the shape of our limited equation and that separates the space of basic variables into safe and failed bridges.

So, in form in FORM the idea is to map the basic variables into another space of variables Y and let us say that there is a one-to-one correspondence between X and Y . So, $Y = T(X)$ where T is a transformation and X is the inverse map from Y , so, these functions are very well defined. In that case the region any region R_x would map to a corresponding region R_y from X to Y and the probability content of the region R_x would equal we have discussed this when we discuss joint distributions would equal the probability content of the region R_y .

So, that is a key element there. So, which basically means that the failure probability would be whereas in the basic variable space we were trying to find the probability that X belonged to R_x . Let us say R_x is the failure region then equivalently we can find out the probability content that Y belongs to R_y . And why would we do that obviously if that computation or that estimation of the failure probability in Y space is considerably simpler that would be an inducement of doing such an exercise.

So, two key questions come here one is. So, what sort of property should Y have where do we want to map from the basic variable space to this new space and what sort of map should we try for. So, these are the two key questions that we are going to answer next.