

Structural Reliability
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Lecture –160
Capacity Demand Component Reliability (Part 08)

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Computing probability of failure

Structural Reliability
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Example A1: two-RV cable reliability problem

We continue with the cable in a suspension bridge made of A36 steel and treat it as a two-RV problem. The yield strength Y (time invariant) is normally distributed with mean strength 38ksi and COV 15% as before. We introduce randomness into the axial load Q and take it to be normally distributed with mean 1000kip and COV 10%. Y and Q are independent.

Cable failure is defined as yield of the section.

Find the cross sectional area of the cable if the target failure probability is 0.001.

$$\{\text{Failure}\} = \begin{cases} aY < Q \\ aY - Q < 0 \\ aY / Q < 1 \\ \ln a + \ln Y - \ln Q < 0 \end{cases}$$

Safety margin, $M = ???$
 choose $M = aY - Q$
 so that Failure = $\{M < 0\}$

M is normally distributed with mean $\mu = 38a - 1000$ and variance, $\sigma^2 = 32.5a^2 + 10^4$

The failure probability is: $P[M < 0] = \Phi(0 - \mu) / \sigma = 0.001 = \Phi[-3.09]$ (equating with the target)

Simplifying, we get $a^2 - 67.1a + 799.5 = 0$ which gives (the valid root as) $a = 51.5 \text{ in}^2$.



In this problem we again meet our old friend the cable under tension but there are some new aspects here. So, let us take a minute to read the problem and then let us solve it step by step. So, as you see we now have a two random variable problem and in addition to Y the load Q is also a random variable now Y and Q are independent and it. So, happens that both of them are normally distributed.

We need to solve the problem not only find the failure probability but make sure that it does not exceed 0.001 by choosing the appropriate cross-sectional area a which is non-random. Now here is an interesting question that there are different ways of describing failure they should be equivalent we could define a Y less than Q we could define a Y minus Q less than 0 we could define a Y divided by Q less than 1 we could describe we could take log obviously taking log would be a problem because a Y and Q also have negative values but this is a general

representation.

Now, which one of them would be easiest to solve or would they give different answers and if so, then can we impose some in variance conditions can we do something. So, the answer is invariant to the way we describe failure. These are questions we are not ready to answer yet but these are important questions for now. We see that because Y and Q are normally distributed it makes most sense to take a failure representation which is a linear combination of normal.

So, Y minus a Y minus Q would be the most logical choice for the limit state function and that is what we do we from that point on it becomes a one variable problem in the cross section area a because we are all putting we are putting all the values of the mean and variance of Y and Q. So, in the end we have to solve the problem that the failure probability is 0.001 and that gives me the answer for a it is 51.5 square inches.

So, this tells us how to not only set up the problem which we have done in the past but now we need to think beyond that and see how to solve it. And here we were lucky the problem could be formulated in a manner that we could solve it in closed form just by estimating the normal cdf.

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Example A2: two-RV cable reliability problem

We continue with the cable in a suspension bridge made of A36 steel, but change the distributions from **normal to lognormal**.
 Y is now lognormal with mean 38 ksi and COV 15%.
 Q is now lognormal with mean 1000kip and COV 10%. Y and Q are independent.

Cable failure is defined as yield of the section.

Find the cross sectional area of the cable if the target failure probability is 0.001.

$$\{\text{Failure}\} = \begin{cases} aY < Q \\ aY - Q < 0 \\ aY / Q < 1 \\ \ln a + \ln Y - \ln Q < 0 \end{cases} \quad \begin{array}{l} \text{Safety margin, } M = ??? \\ \text{so that Failure} = \{M < 0\} \end{array}$$

What if Y- Weibull and Q - Gumbel?
What if area also is random?
No exact analytical solution!

Choose safety margin, $M = \{\ln a + \ln Y - \ln Q < 0\}$

M is normally distributed with mean = $\ln a + 3.63 - 6.9 = \ln a - 3.27$ and variance = $.0998^2 + .149^2 = 0.032$.

The failure probability is: $P\{M < 0\} = \Phi\{0 - \ln a + 3.27\} / 0.18$.

Equating this with the target, we get $\ln a = 3.27 + 0.18 \Phi^{-1}[0.001] = 3.83$ which gives $a = 45.6 \text{ in}^2$.



Here we stay with the same problem but put a little twist this is what we are calling example A2

and if you just read it carefully you will see that this one difference from example A1 is that the distributions of Y and Q are each log normal. So, it is probably going to change the answer but we need to set up the problem properly and solve it. So, let us take a few seconds to acquaint ourselves with the problem statement and then we will proceed with the solution.

So, here again the question as to which failure representation to take because Y and Q are log normal it seems the obvious choice is the last one which is involving log Y and log Q because we know that they are each normal and we very much like to have a linear combination of normal. So, we take the safety margin as I just said and now it's again straightforward we have solved such problems before.

We come up with the statistics of M in terms of a the unknown cross section area and impose the condition that the failure probability is 0.001 and that gives us an answer slightly lesser than the previous case it is 45.6 square inches. Now let us confront this question now because it is it is something we are going to deal with quite a lot is that what if we did not have the benefit of such simple closed form solutions what is the problem formulation or the distributions.

But such that this nice simple analytical solutions were not possible what should we do and we have one example here if suppose Y was viable and Q and Gumbel. What if the area was also random? So, then we have situations where simple linear combination of normal you know is no longer applicable. But we still need to solve such problems and that is why we are going to look at approximate solutions.

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Computing probability of failure - toward FORM

Generalizing:

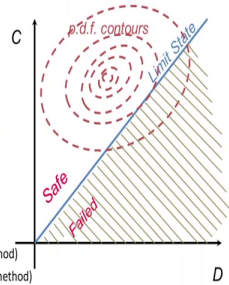
\underline{X} = basic variables, must be defined
 $g(\underline{X})$ = performance fn., must be available at least point wise,
 great if differentiable
 $g(\underline{X}) = 0$ is the limit state eqn. safe vs. failed binary state

$C > D$: Safe
 $C < D$: Failed

Probability content of the failure region:
 $P[\underline{X} \in \bar{\Gamma}_{safe}] = P_f = P(g(\underline{X}) < 0) = \int_{g(\underline{X}) < 0} f_{\underline{X}}(\underline{x}) d\underline{x}$

• **Computation of failure probability:**

- Analytical
 - Exact
 - Approximation -- FORM (first order reliability method)
 - Approximation -- SORM (second order reliability method)
- Simulation-based
 - Brute force -- Direct Monte Carlo



And in particular we are going to look at solutions approaches of the type where we would like to estimate the probability content given a limit state equation in terms of basic variables of arbitrary size and arbitrary distributions. And what we are going to look at in the next few lectures is to be able to compute the failure probability for such a situation analytically exact which we actually did one or two examples approximations the very elegant first order reliability method.

It is more a little more complicated because in the second order reliability method SORM and then simulation based approaches both direct or brute force Monte Carlo simulations which we have alluded to in the past and at least one variance reduction technique the important sampling method.