

Structural Reliability
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Lecture –16
Review of Probability Theory (Part -08)

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Review of Probability

Structural Reliability
 Lecture 2
 Review of
 probability theory

Examples

A laboratory is configured to run four different processes: A, B, C and D. Only one process may be run on a given day, the likelihood being 10%, 50%, 30% and 10%, respectively. Each process may produce harmful effluents - the respective likelihoods of which are 50% for A, 5% for B, 20% for C and 35% for D.

Find the probability that there will be no discharge of harmful effluents in a day.

If harmful effluents have been produced, what is the probability that process C was operating on that day?

Define, $A = \{\text{process A ran during the day}\}$, likewise B, C, D
 $H = \{\text{harmful effluent produced}\}$

Given, $P[A] = 0.1, P[B] = 0.5, P[C] = 0.3, P[D] = 0.1$
 $P[H | A] = 0.5, P[H | B] = 0.05, P[H | C] = 0.2, P[H | D] = 0.35$

Applying theorem of total probability,
 $P[H] = P[H | A]P[A] + P[H | B]P[B] + P[H | C]P[C] + P[H | D]P[D]$
 $= 0.17$
 $\Rightarrow P[\bar{H}] = 0.83$

Required, $P[C | H] = ?$

Applying Bayes' theorem,

$$P[C | H] = \frac{P[H | C]P[C]}{P[H]} = \frac{0.2 \cdot 0.3}{0.17} = 0.353$$

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This problem has to do with environmental engineering. Let us read the problem carefully and identify the basic events. So, let us define a as the process A ran during the day and likewise we can define B that the process B run during the day and likewise for C and D this another important event H which is harmful effluents were produced. So, if we now interpret the probability numbers we would assign in this way the probabilities of A, B, C and D are respectively 0.1, 0.5, 0.3 and again 0.1 and this would be our partition.

So, our partition is A, B, C, D and one way to make sure that you got the partition right is the their each events probability in that partition should add up to 1 and that is what you see is happening here. The next four are conditional events. So, H given A, H given B, H given C and H given D and those probabilities are also stated. So, now we can apply the theorem of total probability and find the unconditional probability of H.

And that turns out to be 0.17 once you do the arithmetic and what's been asked for is there will be no discharge. So, P of H bar which is 0.83. Now the second part of the question asks that if harmful effects of harmful effluents have been produced what is the probability that process C was operating. So, let us interpret the question in terms of probability. So, it is Pp of C given H. So, now H becomes the conditioning event and.

Now we can invoke the Bayes theorem and P of C given H would be P of C multiplied by a ratio which is P of H given C over P of H and we just found out P of H and the other 2 probabilities we already know they are given. So, if you do the numbers it comes down to 0.353 that given harmful effluents have been detected it was process C that produced it as a probability of 35.3%.