

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –159
Capacity Demand Component Reliability (Part 07)

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Computing probability of failure

Structural Reliability
 Lecture 19
 Capacity demand
 component reliability

Example: safe stopping

(a) CASE I & II
NO CONTROL OR YIELD CONTROL ON MINOR ROAD

TABLE 2 Best Data Used in the Application Example (Cases I and II)*

Variable	Extreme Value ^b (AASHTO)	Mean Value ^c	Coefficient of Variation
V_A	80 km/h	64.94 km/h	0.1
t	2.5 s	2.15 s	0.1
f_A	0.30	0.36	0.1
V_d	50 km/h	40.58 km/h	0.1
f_d	0.35	0.42	0.1

*The correlation coefficients are: $\rho_{V_A} = 0.5, \rho_{t_A} = 0.5, \rho_{f_A} = -0.5$, and $\rho_{V_d} = -0.5$. Level grades are assumed.
^bThe AASHTO stopping sight distances are 62.8 m and 139.4 m for design speeds of 50 km/h and 80 km/h, respectively.
^cThe mean values are computed assuming that the extreme values represent the 99th percentile for V_A and V_d ($Z = 2.32$), the 95th percentile for t ($Z = 1.64$), and the 5th percentile for f_A and f_d ($Z = -1.64$).

Demand, S_d = required sight distance (meters)
 $= 0.278V_A T + V_d^2 / [25.4(f_A + G_A)]$
 Capacity, S_{sup} = supplied sight distance
 Safety margin, $M = S_{sup} - S_d$

V_A = initial speed of vehicle on approach A (km/h)
 T = perception reaction time (s)
 f_A = coefficient of friction between road A and tyre
 G_A = grade of road A

From Easa, TRR 1701, paper no. 00-2355



We have already seen this problem when we discussed how to formulate a reliability problem this involves a design of an intersection from transportation engineering. So, we have to make sure that the required side distance is less than the supplied side distance but there are random variables involved. So let us solve this problem with some typical numerical values.

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Computing probability of failure

Demand, S_d = required sight distance (meters)
 $= 0.278V_A^2 T + T^2 / [254(f_A + G_A)]$

Capacity, S_{cap} = supplied sight distance

Safety margin, $M = S_{cap} - S_d$

Variable	Extreme Value ^a (AASHTO)	Mean Value ^a	Coefficient of Variation
V_A	80 km/h	64.94 km/h	0.1
t	2.5 s	2.15 s	0.1
f_A	0.30	0.36	0.1
V_A	50 km/h	40.58 km/h	0.1
f_A	0.35	0.42	0.1

V_A = initial speed of vehicle on approach A (km/h)
 T = perception reaction time (s)
 f_A = coefficient of friction between road A and tyre
 G_A = grade of road A

Given $S_{cap} = 140m$. Define failure as inability to stop within supplied sight distance. (Take $G_A = 0$).

- Case 1. Take only T to be random in road A, and others at design values. What is P_f ? Take: (i) T as Normal, (ii) T as lognormal, (iii) T as Weibull.

Ans: When $V_A=80\text{km/h}$, $f_A=0.3$, $G_A=0$, then $SA=22.2T+84.0$, and $\{Failure\}=(T>2.52s)$

- (i) T - Normal (mean=2.15s, sd=0.215s). $Pf=1-F(2.52;2.15;0.215)= 0.043$
- (ii) T - lognormal. $Pf=1-F(\ln(2.52); \ln(2.15); 0.0998)= 0.055$
- (iii) T - Weibull ($\alpha=2.24, k=12.2$). $Pf=\exp[-(2.52/2.24)^{12.2}]=0.015$

- Case 2. Take T , V_A and f_A as random. T -Normal, V_A -lognormal, f_A - Weibull. How will you estimate collision probability?

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So, on the top left you see the limit state which is $M = S_{supplied} - S_{required}$ all the random variables are defined and all the other constants are defined. So, we and it is also stated that the supplied side distance is known to be deterministically 140 meters. So, suppose only T the time of the perception reaction time is the only random variable and all other parameters are fixed non-random at their design values.

So, we can solve this problem under various assumptions that this T random variable T is normal or log normal or viable and it would be a simple exercise you have done solve these problems already several times. So, if you want to work through them please pause the video otherwise let me present the answers. So, when we fix the non-random parameters at their design values we get a limit state failure as T greater than 2.52.

So, here failure is written in an inverse form typically we have $g < 0$ but here T is greater than 2.52. So, you can think of it as $2.5 - T$ would be our limit state function anyway. So, when T is normal we can solve it and the answer comes to something like 4.3% when T is log normal we can solve it and the answer is a little different when T is viable we can also solve it the answer is again smaller.

So, clearly and we know by now that the distribution of the random variables can significantly

affect the answer in terms of P_f or reliability. But we have a more interesting question here is that what if T is not the only random variable which is what we should expect and what if V_A the initial speed and f_A the friction coefficient they are also random variables and we can define their properties either from data or from the literature or from some reasonable assumptions in either case how will we estimate the collision probability.

It is not a simple cdf calculation anymore there are three random variables involved and there is no way that we can combine all of them into a nice analytical closed form answer. So, what we are hinting at here is that there are situations and which are more common than the otherwise is that we have to resort to approximations we have to come up with estimation techniques for failure probability where an exact solution is not readily available. So, we need approximations and we are going to present them very soon but let us go through one more example.