Structural Reliability
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Lecture –158
Capacity Demand Component Reliability (Part 06)

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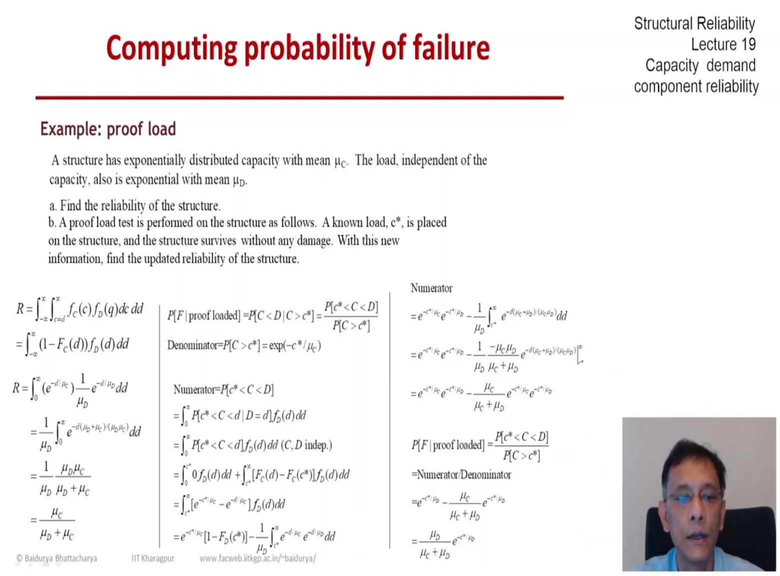

When discussing probability and random variables we did take up an example involving proof loads and saw how the knowledge of proof load as an additional knowledge helps us update the distribution of the capacity. Now let us take that example a little further and estimate the reliability of the failure probability of such a structure. So, let us take a minute to read the problem it has two parts part A is what you see on the screen.

So, we need to find the reliability of a structure whose capacity is exponential and whose demand load is also exponential and they are independent. In part B a proof load has been conducted and we need to find out with this new piece of information the updated reliability of the structure we proceed the same way as we did in defining the reliability couple of slides back as the joint integration of C and D.

So, here we take advantage of the fact that C and D are independent. So, the joint density is the product of the marginal densities. And let us make sure that the limits are written correctly on the double integral. So, d goes from minus to plus infinity all possible values of d but c is restricted to d or higher because we are looking for the reliability. And now we convert that into a one-dimensional integration because the integration of F c becomes one minus the CDF of c evaluated at d and then that integrated with respect to the density of D.

And now we are able to plug in the expressions for the CDF and the density of c and d respectively and if we go through the algebra we can show that the reliability is the ratio of the mean capacity over the sum of the mean demand plus the mean capacity. So, the higher the mean capacity in relationship to mean demand obviously reliability becomes higher and higher. Now let us try to answer part B.

So, here we are looking for a conditional probability as we already have seen in part A that the probability of failure given the proof load that would be my updated failure probability or one minus that would be the updated reliability. So, we can expand that in terms of C and D and that proof load value C star. So, now my failure event is conditioned on C being greater than C star and failure event is C less than D as before.

So, once I write that out as the conditional probability let us take the denominator and numerator one by one. So, the denominator is simply that C is greater than C star. So, that would be the the complement of the CDF of C which is there as you see and now let us look at the numerator. Numerator clearly is the difference of 2 CDF’s however D must be greater than C star. So, let us make sure we put those conditions and we also make use of the fact that C and D are independent.

So, we go through the process separate the integral into two regions 0 to C star and C star to infinity and once we go through the algebra it's a little involved. So, let us let us continue and the answer will be given in you know next step that is the original answer but it is now modified by the probability that exponential minus C star over mu D. So, that is the modifying term to the failure probability. So, now my failure probability is less than what it was before.