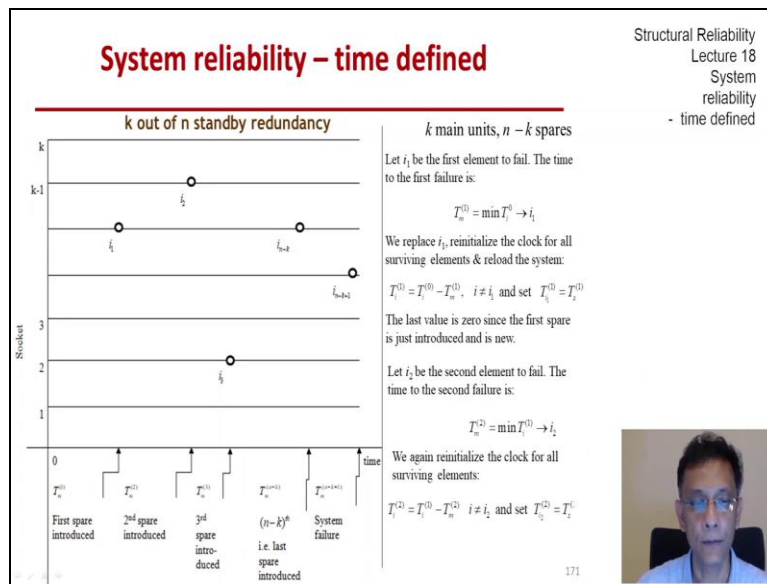


Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –152
System Reliability - Time Defined (Part - 08)

(Refer Slide Time: 00:27)



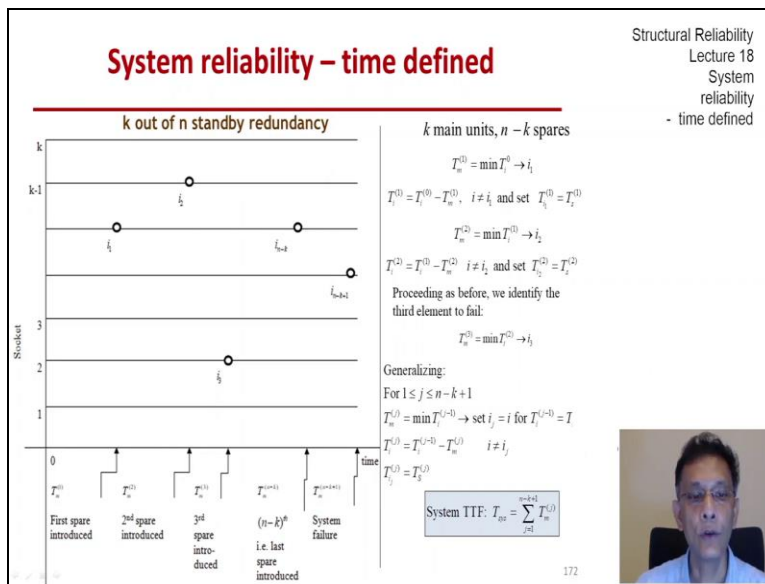
Thus far we have looked at various kinds of active systems now before we end the lecture let us just spend a few minutes on looking at standby systems. So, the general layout of the standby architecture is something like this that there are k sockets numbered from 1 through k that you see on the very left are the vertical axis and the x-axis is the time axis and there we indicate all the different times in which spares are introduced.

So, the system is formed of k main units and they are all identical and n minus k space so let us start the clock. So, let i 1 be the first element to fail the time to the first failure is the smallest one. So, the smallest of all the T i's is that first failure that is the element identified as i 1 and we note that quantity tm superscript 1 in parenthesis. So, that is the first time that a spare has been introduced and what we do is we replace that element we re-initialize the clock for all the surviving elements and reload the system.

So, this is what I do this is what we do T_i superscript 1 is now the clock after the first failure. So, that is reset by it is just relocated with t_m one subtracted for all of them except the element that failed and the element that failed we just give the we just assign the first spares lifetime T_s superscript 1 to that new element that goes into the same socket i_1 . So, this continues let i_2 be the second element to fail and the time to the second failure is t_m superscript 2 and that is how we identified i_2 .

The second element to fail and then we replace that with a spare with live T_s superscript 2 and we reset and reinitialize and put the system back online again and. So, that gives me T_i superscript 2 for all of them except i_2 now it is failed it is now the life of the second spare and this continues.

(Refer Slide Time: 03:42)



So, what we have done. So, far is we have obtained T_M superscript one we have obtained T_M superscript 2 we have put in two space and then we get the third element. So, it is i_3 and we can generalize this process and that is what you see at the bottom right corner is for all j from 1 to $n - k + 1$ we find the T_m 's and we keep replacing until we are we all the spares are exhausted. So, the system time to failure in this scheme is the sum of all the T_m 's that we have identified.

(Refer Slide Time: 04:39)

System reliability – time defined

The standby parallel system

(1 active, n-1 standby elements)

$$T_{sys} = T_1 + T_2 + \dots + T_n$$

$$\mu_{sys} = \sum \mu_i$$

Simple standby redundancy is better than active parallel redundancy since:

$$\max T_i \leq \sum T_i$$

regardless of whether the component TTFs are dependent or not (beware switching failure).

2 element standby parallel

T_2 starts only when the first unit fails.



Here T_1, T_2 are independent from load sharing point of view, but may not be from common cause if it existed.

$$R_{sys}(t) = P[T_1 + T_2 > t] \quad R_{sys} = \sum_{t=0}^{\infty} P(T_2 > t - t_1) P[T_1 = t_1]$$

$$= \int_0^t R_2(t - t_1) f_1(t_1) dt_1 + \int_t^{\infty} (0) f_1(t_1) dt_1$$

$$= \int_0^t R_2(t - t_1) f_1(t_1) dt_1 + R_2(t)$$

$$R_2(t') \geq R_2(t) \text{ for any } t' \leq t.$$

$$R_{sys} \geq R_2(t) \int_0^t f_1(t_1) dt_1 + R_2(t)$$

$$= R_2(t)(1 - R_1(t)) + R_2(t)$$

$$\text{or } \boxed{R_{sys}(t) \geq R_1(t) + R_2(t) - R_1(t)R_2(t)}$$



We could now just look at a very simple standby parallel system with one active and n - 1 standby element. So, the system time to failure is simply the sum of all the individual times to failure because once the first one fails the second one takes over. So, all the TTFs are added to get the system time to failure whether the T's are dependent on or not does not matter the mean is the sum of the means this mean the entity for the system is the sum of the individual entities.

Now again whether the T's are dependent or not the individualities the system time to failure will always be better than we if we had put all of them active together. So, by which I mean that this max T I which would be the time to failure had it been an active system that is always less than the sum of all the T's because the max T i is just one component in that sum. So, if we can afford it if there is no overloading if there is no stressing of elements obviously it makes sense to not put all of them into service if we have one or more spares.

Let the first one fail then the second one can take over and so, on until we are out of all the n - 1 spares obviously what you see there is we have to be careful about switching failure. Now let us look at this just with a two element parallel system and we will we will look at T 1 and T 2 the system reliability is now just T1 + T 2 greater than little t the probability of that and we can employ the theorem of total probability as we have been doing a lot.

And that gives me the integral which is integration from 0 to t of R 2 t - t 1 times the density of t

1. So, that is the contribution of the second element. So, that is the contribution of spare the first one is $R_1(t)$ and it is easy to show and which we have seen the proof in the bottom of the screen is that that the system reliability in this standby situation without any switching failure of course is always going to be better than the active system consisting of the same two elements. We can bring in switching failure now.

(Refer Slide Time: 07:58)

Structural Reliability
Lecture 18
System reliability
- time defined

System reliability – time defined

The standby parallel system
(1 active, n-1 standby elements)

$$T_{ss} = T_1 + T_2 + \dots + T_n$$

$$H_{ss} = \sum H_i$$

Simple standby redundancy is better than active parallel redundancy since:

$$\max T_i \leq \sum T_i$$

regardless of whether the component TTFs are dependent or not (beware switching failure).


2 element standby parallel
 T_2 starts only when the first unit fails.

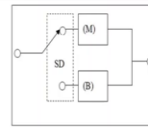
Here T_1, T_2 are independent from load sharing point of view, but may not be from common cause if it existed.

$$R_{ss}(t) = P[T_1 + T_2 > t] = \sum_{t_1=0}^t P(T_2 > t - t_1) P(T_1 = t_1)$$


$$= \int_0^t R_2(t - t_1) f_1(t_1) dt_1 + \int_0^t (0) f_1(t_1) dt_1$$

$$= \int_0^t R_2(t - t_1) f_1(t_1) dt_1 + R_1(t)$$





Standby configuration with switch: $R_{ss}(t) = R_{ss} \int_0^t R_2(t-x) f_M(x) dx + R_M(t)$



©Baidya Bhattacharya | IIT Kharagpur | www.facweb@iitkgp.ac.in/~baidya/ 174

So, now we have a switch a switching device which at first is connected to M the main unit and B is the backup unit. So, once M fails the switch brings the unit B online and hopefully this switching over are seamless but switches do fail switches do age. So, the longer the switch waits maybe an aging happens. So, there is a possibility that the switch may not work as intended. So, if the first element fails if the main unit fails the backup never comes online.

So, that is a consideration that we have to keep in mind and that might change the whole picture that might actually be an argument in favor of not having a standby parallel system but having both of them active and that is why you see the RCST augmented by RST the reliability of the switching device which in a more complicated situation could also be a function of time but here it is not.

So, it is out of the integral. So, you see the way that it modifies the reliability due to the backup and the R_M or R_L in the upper block is the main unit is reliability. So, if there is a switching

failure issue there is no telling which configuration would be better.


(Refer Slide Time: 09:47)

System reliability – time defined

Structural Reliability
Lecture 18
System
reliability
- time defined

```
graph TD; A[Fire prevention system fails] --- B{AND}; B --- C[Sensors fail]; B --- D[Power fails]; B --- E{Sprinkler fails}; C --- F{AND}; F --- G((Sensor 1 fails)); F --- H((Sensor 2 fails)); D --- I{AND}; I --- J{Mains fail}; I --- K{Diesel generator fails}; E --- L{Switching failure};
```

- A fire prevention system consists of three subsystems: sensors, power and sprinkler as shown in the above figure. Every basic and undeveloped event is independent of one another.
- The point availability of the mains power at any time is 80%.
- The generator is switched on only if main power fails. It deteriorates in this standby state and its TTF is exponential with mean 10 year. In addition, the switching failure probability is 10%.
- Of all the subsystems, only the sensor system needs to be "on" all the time. Each sensor's TTF is exponential with mean 3 years. Only one is enough to detect fire.
- The sprinkler subsystem does not deteriorate and has a switching failure probability of 5% when required.
- The system is installed in new condition. When should inspection be scheduled if the system reliability is not allowed to fall below 90%?



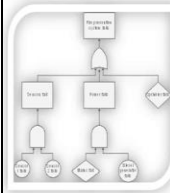
©Balabunga Bhattacharya - IIT Kharagpur www.facebook.com/balabunga/ 175

Let us solve one example to end this and we have looked at this before when we did solve some problem in system reliability uh. So, here we are back to the same situation but now there is explicit time dependence in some of this unit's failure. So, the text in red indicates to that the generator has a TTF exponential TTF would mean 10 years and then it has a switching failure probability of 10% as before the census are now exponential with mean 3 years.

And so, and the water system does not have any time dependence but the question now is that when should inspection be scheduled if we do not allow the reliability to fall below 90%. So, let us lay out the outline for the solution.

(Refer Slide Time: 10:53)

System reliability – time defined



- A fire prevention system consists of three subsystems: sensors, power and sprinkler as shown in the above figure. Every basic and undeveloped event is independent of one another.
- The point availability of the mains power at any time is 80%.
- The generator is switched on only if main power fails. It deteriorates in this standby state and its TTF is exponential with mean 10 year. In addition, the switching failure probability is 10%.
- Of all the subsystems, only the sensor system needs to be "on" all the time. Each sensor's TTF is exponential with mean 3 years. Only one is enough to detect fire.
- The sprinkler subsystem does not deteriorate and has a switching failure probability of 5% when required.
- The system is installed in new condition. When should inspection be scheduled if the system reliability is not allowed to fall below 90%?

Say $t = 1\text{yr}$

$$T_G \sim \text{Exp}(1/10\text{yr}), R_G(1\text{yr}) = \exp(-1/10) = 0.90$$

$$R_{\text{power}}(1\text{yr}) = 1 - (1 - 0.8)(1 - 0.9 \times 0.9) = 0.962$$

$$T_s \sim \text{Exp}(1/3\text{yr}), R_s(1\text{yr}) = \exp(-1/3) = 0.717$$

$$R_{\text{sensors}}(1\text{yr}) = 1 - (1 - 0.717)(1 - 0.717) = 0.92$$

$$R_{\text{sprink}}(1\text{yr}) = 0.95 \text{ as before}$$

$$R_{\text{sys}}(1\text{yr}) = 0.962 \times 0.92 \times 0.95 = 0.84$$



We will just give the hint and then if you are interested you might complete the problem. So, let us say T is one here. So and let us see if an inspection at one year will be adequate or not. So, the generator has an exponential distribution as we said. So, its reliability is can be found roughly 0.9. So, the power system has a reliability of 0.96 the sensor system has two sensors each with an exponential TTF with mean 3 years.

So, the sensor system reliability is 0.92 at the end of one year and the water system has no change its reliability is 0.95 as we had done in the previous reincarnation of this example. So, then because they are all independent the reliability of this system is the product and that is 0.84 clearly it is less than 90%. So, one year is not enough we need to schedule the inspection earlier how early that you can solve the most straightforward method is just to solve it at relatively and then you can select at the optimal time to schedule the inspection.