Structural Reliability
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Lecture –150
System Reliability - Time Defined (Part - 06)

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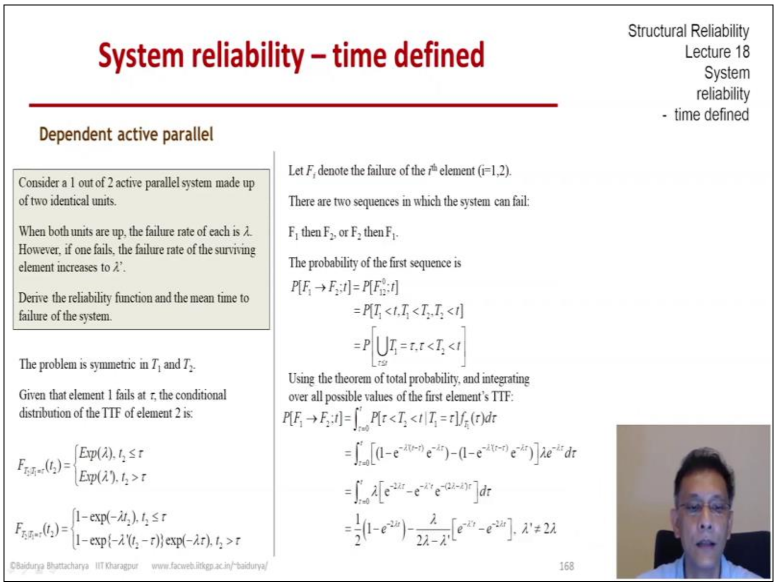

After looking at systems made up of independent elements in various configurations let us switch gears and look at the simplest possible system with dependent elements and that is what you see dependent active parallel with 2 just 2 elements. So, it is a one out of 2 active parallel system the 2 units are identical they are also exponential in nature meaning that their time to failure is exponentially distributed.

When both units are up the failure rate of each is lambda however if one fails because the other one the surviving one has to bear a higher load presumably its failure rate increases to lambda prime. So, with that information we need to find out the system reliability and as an added item the mean time to failure. So, let us take it step by step we need to we need to recognize in the beginning that this problem is symmetric in T 1 and T 2 and a fact which should be useful very soon.

So, let us say that one fails first and we know that then the distribution of T 2 changes from 1 governed by lambda to 1 governed by lambda prime. So, let us say T 1 fails at an arbitrary instant tau. So, before tau T 2 is same as before with rate lambda exponential with red rate lambda and after tau T 2 is now another exponential with rate lambda prime. So, if we want to expand these exponential forms we have to do it carefully before tau there is no problem T 2 has the exponential form with lambda element 1 is still working.

But when element 1 fails then we have to develop the second line of that F T 2 given T 1 carefully the first exponential term basically says that the clock is reset. So, t 2 minus tau is the resetting with a new rate lambda prime but the second factor that exponential minus lambda tau that comes at the end of that line that is to ensure or acknowledge the fact that t 2 can only do what it is supposed to do after element one fails is if it itself survives up to that that time. So, the probability that it survives element 2 survives up to tau is that second factor exponential minus lambda tau.

So, now let us just put some more events in the in the mix. So, F i is the failure of element i. So, we have F 1 and F 2 and there are 2 sequences in which the system can fail and that is where the symmetry is going to becoming helpful. So, either F 1 then F 2 or F 2 then F 1 and the problem is symmetric. So, we will just take the first sequence. So, F 1 fails first then F 2 and that is what on the left the conditional distribution actually pertains to that sequence F 1 then F 2.

So, let us expand the event that F 1 then F 2. So, that basically means that t1 is obviously less than small t but more importantly capital T 1 is less is less than capital T 2. So, element 2 fails second that is very important. And the third event in that line T 2 less than little t is that the second element also fails and that is how at time t the sequence happens. So, F 1 then F 2 and system fails at time t now we can bring in a partition in tau and we are going to use theorem total probability next and that should be obvious by now.

So, we fix T 1 at all possible values tau up to t because the system fails at T and then we combine the limits on T 2 on capital T 2 on the left it is tau and on the right it is T. So, next we apply theorem of total probability and. So, the integral looks like this the first term p within brackets that is conditioned on T 1 taking value tau and then the density of T 1 at tau and the conditioned event is T 2 the random variable T 2 is between tau and small t.

So, if we do the algebra carefully it is the difference of the CDFs of T at those 2 different times and the answer after some algebra comes down to what you see on the screen and if lambda prime is twice lambda then we have to have a slightly different form which you will see in the next slide.

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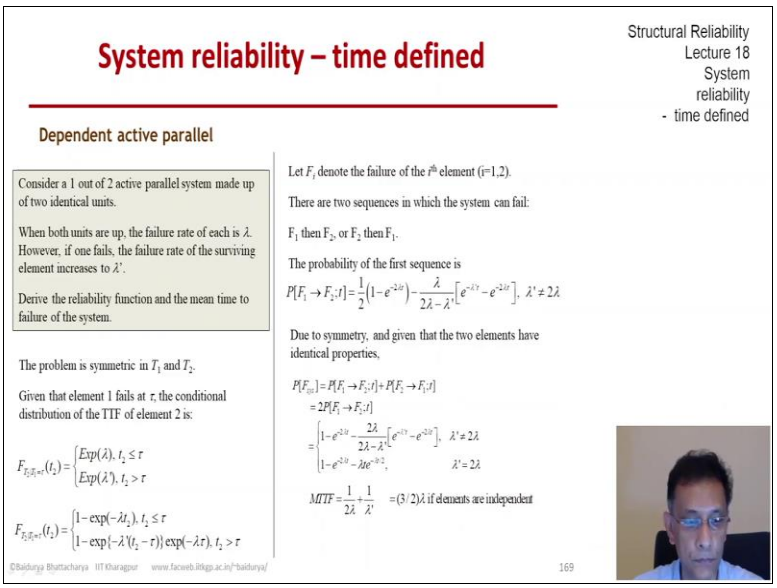

So, F 1 then F 2 that sequence has the probability that we just derived now this is where the symmetry is going to be useful that the 2 sequences are equally likely. So, the F sys the system failure event is the union of F 1 then F 2 or F 2 then F 1 and they are obviously mutually exclusive. So, we can add the 2 probabilities and it is twice because of the symmetry and that's what you see in the end one special case is lambda prime is twice lambda.

So, you have a slightly different form there and then let us find the second task that we were given is the find the mean time to failure of the system and we can do the integration the area under the reliability curve. So, 1 - p of 6 is R sys. So, we integrate that and the MTTF comes to 1 by twice lambda + 1 by lambda prime. And again we looked at the case where the 2 units are independent in an active parallel system and we remember that the mean then was 1.5 lambda sorry it was 1.5 the element mean.

So, it would be. So, if lambda prime and lambda were equal it would be 3 by 2 times 1 by lambda there is a typographical error there. So, it is 3 by 2 times 1 by lambda.