Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –149
System Reliability - Time Defined (Part - 05)

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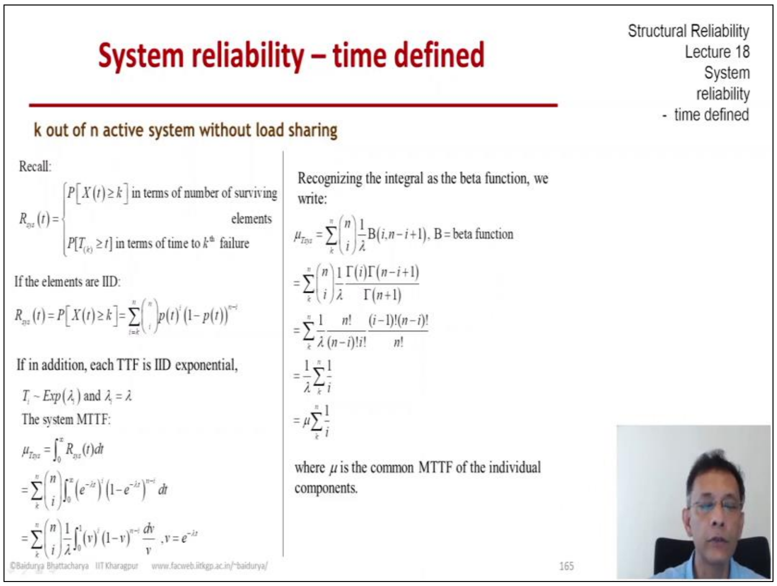

After series and parallel systems let us look at the general case of k out of n active system but we are not talking about load sharing yet. So, all the elements of the system are mutually independent. So, recall that we discussed at the beginning of this lecture that the system reliability is equivalently stated as the probability that at least k systems are up I am sorry k units are up of the system at time t or t sub parenthesis k. So, the kth order statistics is at least a t or higher.

So, now if the elements are IID which is what we assume for this part we have a simple binomial representation of the system reliability and you can clearly identify the binomial sum the sum of the binomial PMFs i going from k to n at least k your elements must work. So, now if in addition if all the TTFs are exponential and independent and IID, so, they have the same lambda. So, under that simplifying assumption we can find the system reliability and the system MTTF the mean time to failure.

So, the mean time to failure as always is the area under the reliability curve and we can go through the steps and we write out the exponential PDF out the exponential following reliability for each element. So, that comes to by substituting nu for exponential - lambda t when lambda is the individual parameter of the TTF it turns out that we come to a form which is the beta function and the beta function is the ratio of involving 3 gamma functions.

So, we have gamma i gamma n + 1 - i and gamma n + 1 in the denominator and gamma function for integer values is nothing but the factorial. So, gamma n is n - 1 factorial with that definition we can come with the final solution which is the mean time to failure for the system is the mean time to failure of the element times an amplification factor which is sum of 1 over i going from k to n.

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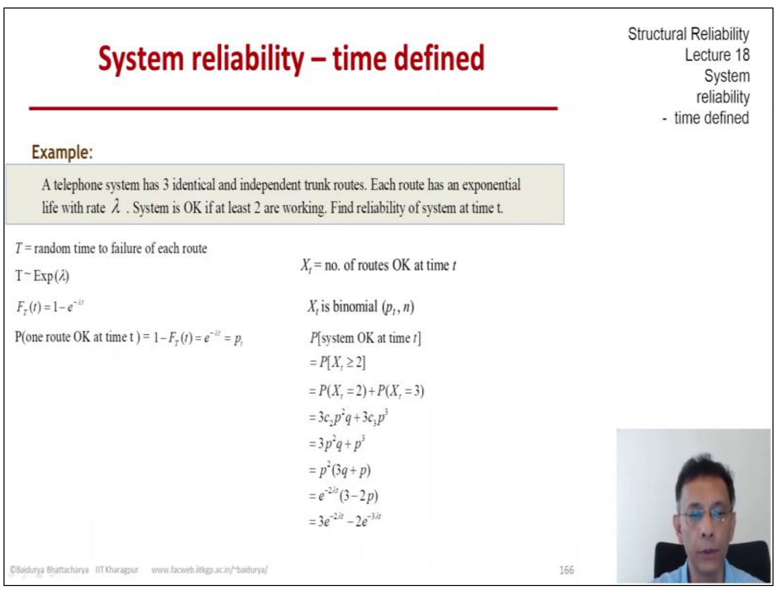

So, for such k out of n active systems with independent components let us solve a couple of examples the first one let us take a minute to read the problem. So it is a 2 out of 3 system and each time to failure is exponential in nature. So, the probability that any one of the elements is working at time t is exponential - lambda t which we call p sub t, so, that is the success probability for each element.

Now let us define X of X subscript t as the number of roots that are at time t. So, X t is binomial as we saw in the previous slide with parameters p t and n and the system is okay at time t is the probability that X t is at least 2. So, with 3 elements this is x t equals 2 or x j equals 3. So, we can add the 2 probabilities and if we do the algebra it just comes to twice p square q plus p cubed where q is 1 - p and the final solution is given as a difference of 2 exponentials.

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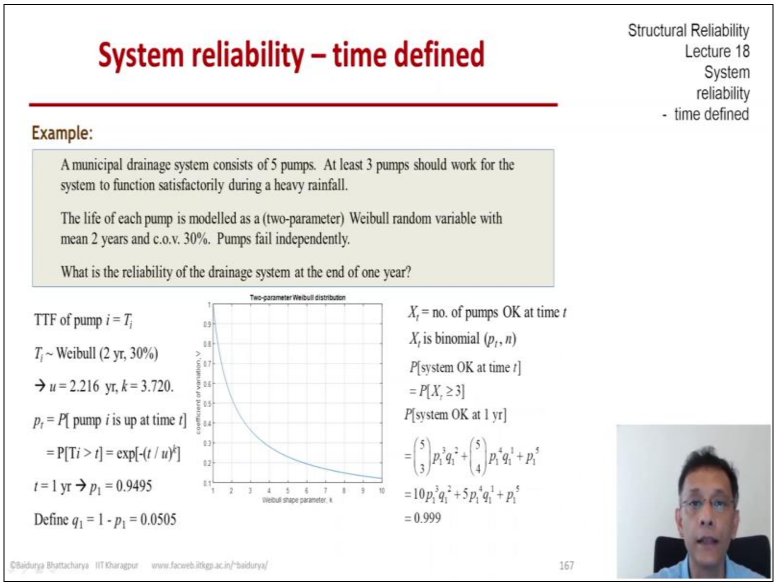

The next problem is not an exponential distributed element time to failure it now has a viable distributed time to failure. So, let us take a minute again to read the problem and then we will solve it step by step if you would like to solve it yourself please pause the video. So, what we have is a 3 out of five system each ttf as i said is viable distributed. So, let us write that out. So, T i viable with a mean of 2 years and a COV of 30%.

So, we need to find the 2 parameter variable parameters. So, we did u and k and we have looked at this graph before. So, this is a convenient way of finding k from the COV for our 2 parameter rival and for this set we find that k is 3.72 and u is 2.216. So the viable complementary CDF the viable reliability function is exponential - of t by u to the power of k. So, plugging in the values of t and u and k we just found out at one year this p of t which we are calling p 1 is 0.9495 we will also need its compliment q.

So, let us define q 1 as 1 – p 1. So, with all this information we are now ready to find the system reliability. So, as we did in the previous example X t is the number of pumps that are okay at time t. So, what we know now is that x t is binomial with 2 parameters p t and n for this example n is 5 and we just found out pt which is 94.95%. So, the system is okay at time t is it is a 3 out of 5 system. So, X t is at least 3. So, we need to sum the 3 probabilities X t equals 3 then X t equals 4 and X t equals 5.

So, that would be the sum of the 3 binomial PMFs and if I have done the algebra correctly we should get an answer of 0.999. So, which is a very highly reliable system provided obviously the pumps actually operate and fail independently this if you compare with if all 5 pumps had to work then we would have a much lower reliability because we would only have p to the power 5 which is about 77%.

So, this is the benefit of adding redundant elements and as I said if the elements indeed operate independently there is no common cause failure there is no overloading if successive pumps fail then we can expect an increase of reliability from 0.77 to about 0.999.