Structural Reliability
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Lecture –147
System Reliability - Time Defined (Part - 03)

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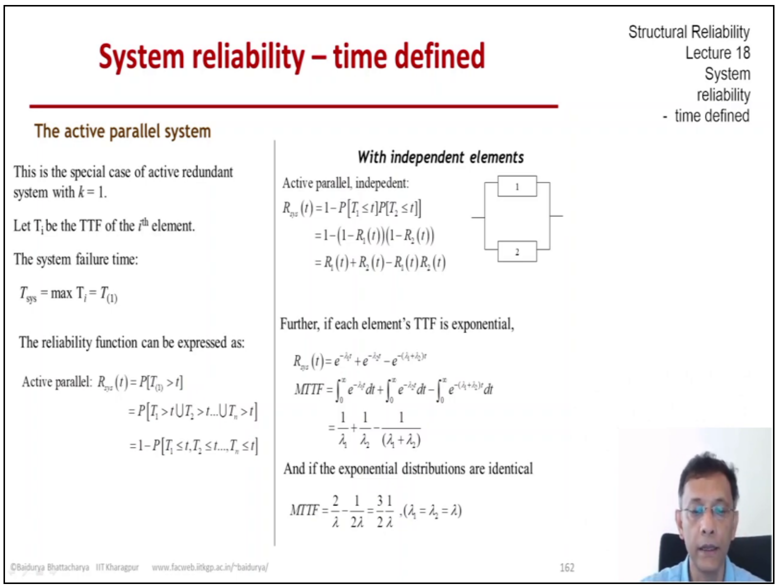

The active parallel system it is a special case of the k out of n redundant system with k equals one. So, let T i be the time to failure of element i as before. So, the system failure time in terms of the individual T i's is the maximum of all of them and in our current convention it is T subscript parenthesis 1. So, the reliability function of such a system would be the probability that that max T i that T subscript parenthesis one is greater than T and now we can write that in terms of the individual TTF’s in two equivalent ways is that at least one of them has to be greater than T because all we want is the maximum to exceed small t.

So, we could have at least one of them or we take the complement and one minus the probability that all of them are less than or equal to t. So, that would be our way of expressing the system reliability function in terms of any arbitrary T. Now let us see what this would look like with independent elements. So we start with the simple two element system that we did in the earlier series case. So, for the active parallel system with two independent elements the system reliability is 1 minus P that T 1 is less than or equal to T times P of T 2 is less than or equal to t. So, that ends up after the algebra as R 1 of T plus R 2 of T minus the product.

So, when we have two independent units in parallel. So, now if each element's TTF is exponential which is as we have discussed often? A popular choice in the reliability community although not always a correct choice but if they are independent exponentials then the system reliability is exponential minus lambda 1 T plus exponential minus lambda 2 T minus the exponential of minus lambda 1 plus lambda 2 times t.

So, that comes directly from the exponential reliability for the components the mean time to failure we will use the property that the MTTF is the integral under the reliability curve. So we integrate all those terms in the system reliability curve and we find that it is a simple integration the mean time to failure is 1 by lambda 1 plus 1 by lambda 2 minus 1 by lambda 1 plus lambda 2. What if now these were not only independent exponentials but identical so the lambdas are equal in that case the mean time to failure would be one and a half times the elements mean time to failure 3 by 2 times 1 by lambda.

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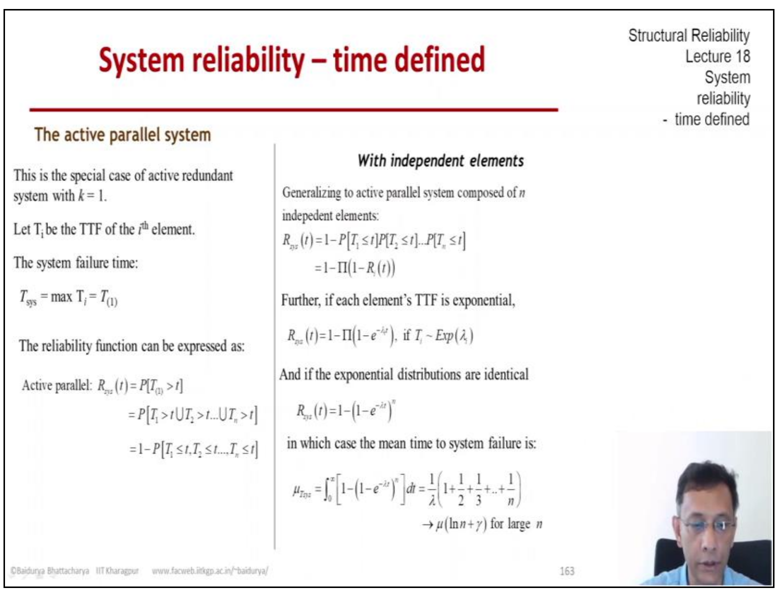

Now let us generalize this instead of two elements let us have n elements all independent of each other. So, the system reliability is 1 minus the product of 1 minus R i at time t. So, R i is the reliability of the individual components. Let us proceed as before if each element's time to failure is exponential in nature then we can bring in the exponential form and the system reliability is 1 minus the product of 1 minus exponential minus lambda i to the T lambda i being the element parameter in the exponential distribution.

So, if now the exponentials if all these TTFs are identical not only independent but identical so all the lambdas will be the same then the system reliability comes down to 1 minus of 1 minus exponential minus lambda T whole to the power of n. And then as we did for the two unit case we can find the mean time to failure and that would be just the integration of this entire reliability function from 0 to infinity.

And we could do the algebra and we could get something in terms of the individual mean time to failure times term that depends on the size of the system n and wireless number of gamma that you that you see which is 0.5771. So, that gives an idea what benefit we are getting by increasing number of elements as far as the mean time to failure is concerned.