Structural Reliability
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Lecture –146
System Reliability - Time Defined (Part - 02)

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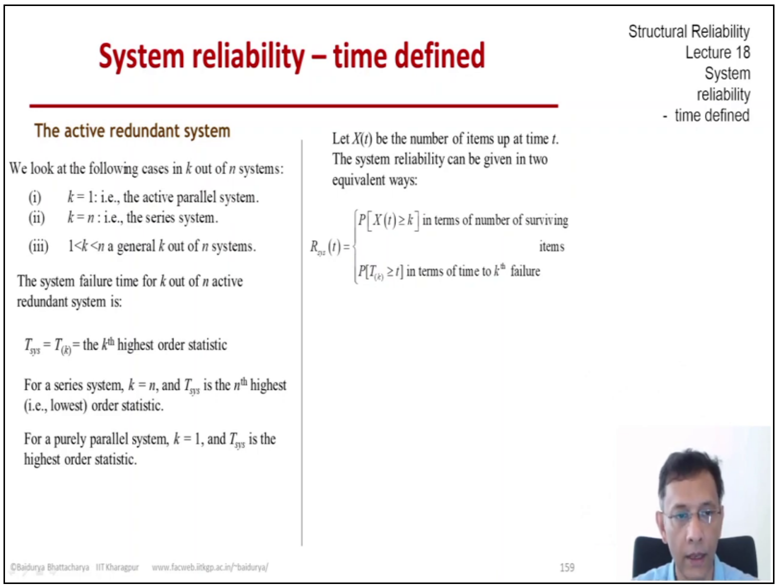

The k out of n active redundant system. So, an active system is where all the components are loaded fully to whatever they need to bear. So, when we mention a k out of n system it means that at least k units must work. So, when we have k equals 1 it is the classical active parallel system we have looked at that briefly before when k is equal to n we have the classical series system all elements must function we have looked at that also and for any other k it is a general k out of n system.

So, we can define the time to failure of all these n elements. So, if we ordered them if we ordered them from highest to lowest. So, T subscript one within parenthesis is the highest and T subscript n in parenthesis is the lowest. So, T n is the weakest the first to fail and T 1 is the strongest and it fails the last. So, for a k out of n system the system time to failure is given in terms of this individual n element time to failures.

So, for a series system T cis is equal to T subscript n in parenthesis and for a purely parallel system the system time to failure is T parenthesis 1 in the subscript. So, there are two ways in which can in which we can define the reliability. So, let x be the number of systems the number of items that are up at time T. So, X of t is the number of items that are up at an arbitrary unit of instant of time T and the R sys the system reliability we could give in two ways that the system time to failure exceeds little T that is equivalently P of X t greater than or equal to k.

So, at least k units are up or the k^{th} order statistic t subscript k within parenthesis that is greater than or equal to t. So, we could define the reliability system in these two equivalent ways.

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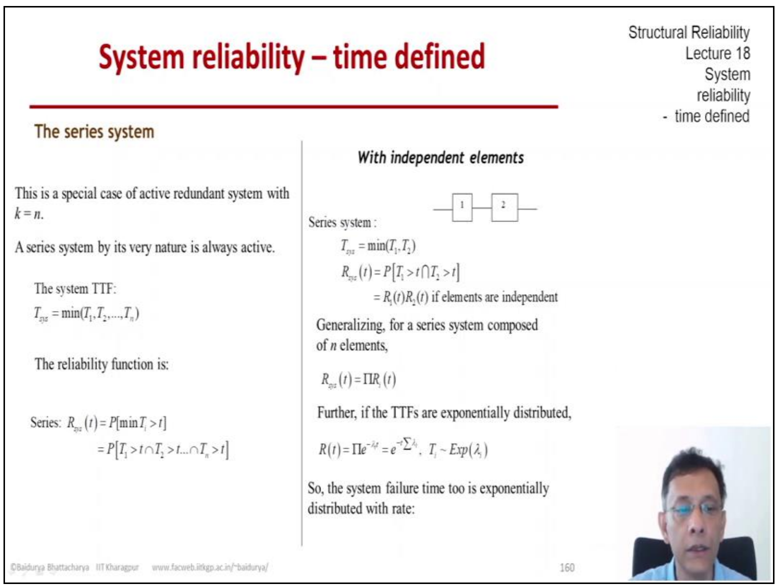

We then start with the series system this is as I said a special case when k is equal to n. So, all units must work and by its very nature just to emphasize that a serious system is always active. The system time to failure is the minimum of all the element times if you ordered them it is the smallest one and the reliability of the system is that the minimum of these element times to failure would exceed the time instant in question.

So, R sys T is the probability that T 1 greater than T and T 2 greater than T all the way up to T n greater than t. So, when we have i in the subscript without parenthesis it is just the time to failure of element i and when we have i within parenthesis in the subscript it means it is ordered. So, it is order statistics k and our convention here is a T parenthesis 1 in the subscript is the maximum the highest one and T subscript n is the shortest one the smallest one.

Now what would happen if these elements were independent we start with a simple two-element system? So, if we have two elements in series we have looked at such comp such systems before. So, T sys is minimum of T 1 and T 2 and R 6 at time T is the probability that T 1 is greater than small t and T 2 is greater than small t now if they are independent then R sys is R 1 at T times R 2 of t. So, all these reliabilities component reliabilities are multiplied with each other to give the system reliability which we have already seen in other contexts.

So, generalizing this R sys t for an n element series system is the product of all the element reliabilities and then if all the TTFs are exponential. So, then we would have the product of these exponentials. So, R of our system of at time T is exponential minus T times the sum of all the lambdas and that is because of the exponential nature of the TTF and so, because of the form of the system reliability clearly the system is also exponentially distributed.

The system time to failure is also exponentially distributed and with a parameter which equals the sum of all the individual lambdas. So, exponential TTFs for each element if they are independent gives rise to an exponential system TTF. And finally if all the elements are not only independent but identical then the system parameter lambda is just n times the element parameter. So, the mean is divided by n. So, the system mean time to failure is the element mean time to failure divided by n.

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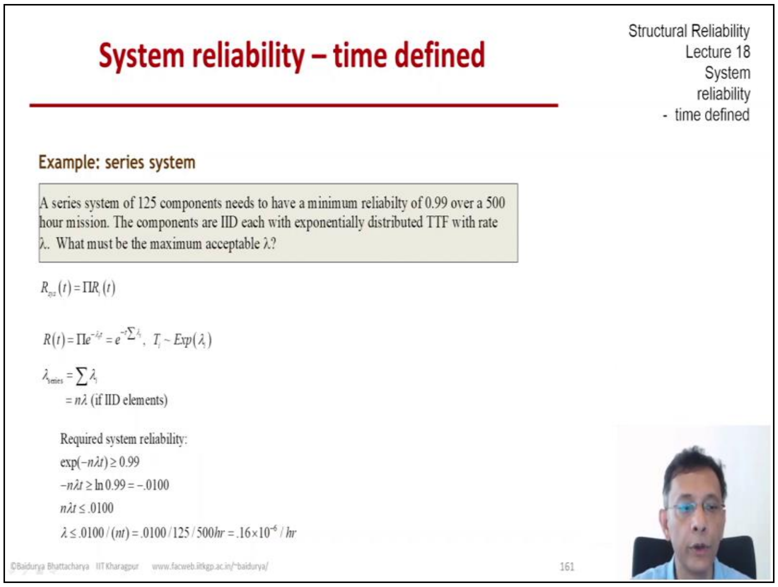

Let us do an example and let us say that we have 125 components which build up a system in series and the time reference in question the mission time is 500 hours. So, we need to find what the element lambda will be the element failure rate will be if the system has to have a required reliability of 0.99. So, let us set this up the system reliability is the product of the element reliabilities and because they are exponential the system the system liabilities of the exponential nature that you see where the sum of all the parameters is the system parameter and that system parameter is equal to n times lambda if all the elements are identical and independent obviously.

So, then what is required is the required system reliability with at least 0.99. So, just doing the algebra from this point on we have n lambda t should be at most 0.01 and putting the values of n and t we arrive at the maximum allowable failure rate for a component at 0.16 per million hours.