Structural Reliability
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Lecture –144
Component Reliability - Time Defined (Part - 23)

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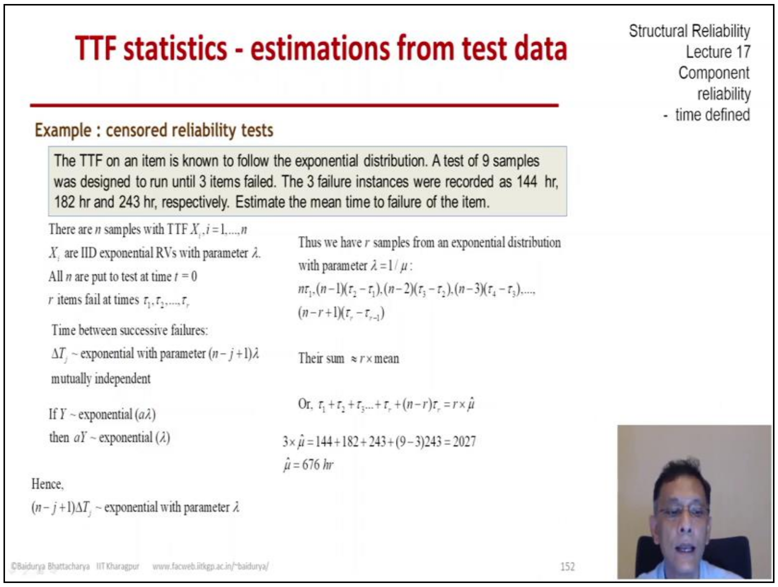

We will be able to sort this out if we can answer this question which is if y is exponentially distributed with parameter a lambda then which random variable is distributed exponentially with lambda. So, if we can answer that question we can make all the previous that those delta T js IID also by maybe doing some little bit of scaling. So, let us start with the basics. So, if y is exponential with parameter a lambda then the CDF is 1 minus exponential of minus a lambda times y.

So, then if we were to divide y by a then the right hand side becomes 1 minus exponential minus lambda y then we could interpret that as a y is less than little y less than or equal to little y uh. So, clearly then what is exponential with parameter lambda it is a y a times capital Y is the random variable that we are looking for. So, with that in mind that if y is exponentially a lambda then a y is exponential lambda we can fix that that non-identical problem that we just mentioned.

So, all the T js as I said are exponential with parameter n - j + 1 lambda they are mutually independent and we already know if y is exponentially a lambda then a y is exponential lambda hence n - j + 1 delta T j has to be exponential with parameter lambda. So, now we have scaled all these exponentials to have the same parameters. So, that gives us a way forward. So, then we can have our samples from an exponential distribution with the same parameter lambda our lambda is 1 by mean.

So, what are these our samples the r samples are n times tau 1 n - 1 times tau 2 - tau 1. So, all these differences are basically the time between failures. So, delta T is the time between successive failures. So, that is how we have entire one which is the time to the first failure n 1 n - 1 times tau 2 - tau 1 which is the time between the first and second failures n - 2 times tau 3 - tau 2 which is the time between second and third phase and so on until n + r + 1 tau r - tau r - 1.

So, each of these terms is a realization of an exponential random variable with the same parameter lambda. So, their sum will be r times mean and here we are going towards an estimation problem which is a bit too simple we are not going for method of maximum likelihood or interval estimation as we have said but we want to get a good answer quickly. So, this simplifies to all that all the tau 1 through r the sum of all tau 1 all the up to tau r and then n - r times tau r.

So, the last recorded failure time that is equal to r times the estimated mean and if now we plug in the three values r is equal to three in this example we get a mean an estimated mean of about 676 hours. So, this is this is quite good uh. So, what we did was we just had three failures and three recorded instances we started with nine samples we did not wait for all nine of them to fail. And now because we knew that the time to failure is exponential for each of them we were able to go through the mathematics and come up with a good estimate of the mean time to failure.

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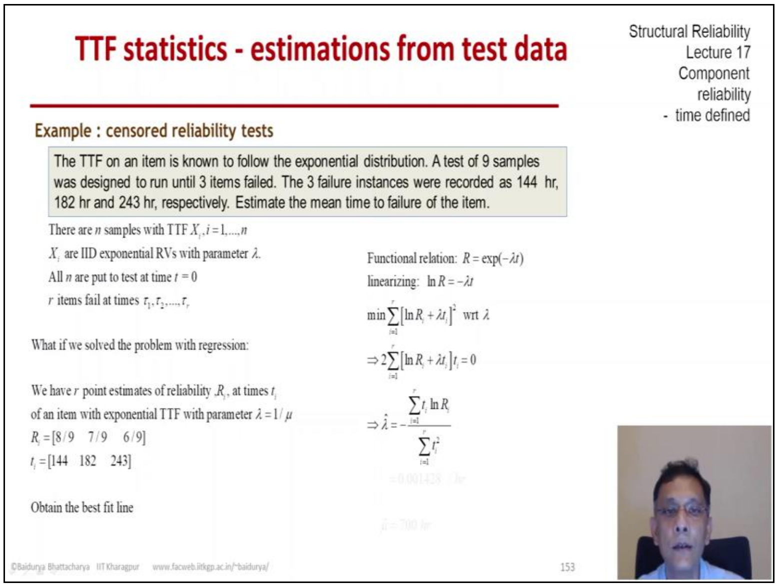

Now we could wonder that with this problem statement that we started with what if we instead of going with this independent exponentials which by the way would lead to a gamma distributed or an Erlang distributed random variable for the sum and we could show later on that this estimate is biased because the way it is been computed but we will not go into those details of more advanced statistical concepts.

But let us come back to this and what if we did not go through all that trouble and we just solve this problem with regression what do I mean by that I mean is that we have basically what we have is our point estimates our number of point estimates of the reliability r i at times t i. So, three times have been recorded we have three reliability estimates corresponding to that of an item with exponential TTF that is important that is very helpful.

So, can we not estimate lambda from just this information can we not fit a curve can we not do regression analysis. So, we have r 8 by 9 7 by 9 and 6 by 9 and the corresponding T's are 144 182 and 243. So, we need to obtain the best fit line between these three points uh. So, what is the step? So, we will just go step by step as we do in a linear regression analysis. So, we have a function relationship which is r equals exponential minus lambda T we need to linearize and that is log r is minus lambda T and then we just minimize the squared error between the theoretical value and the observed value.

So, that would be minimize sum of the square deviations there is a plus sign there because of the negative lambda T at the previous line. So, we need to minimize this sum with regards to lambda. So, now we are in familiar territory we can get the expression which leads to this minimization and it turns out that the estimated lambda would be the ratio of the sum of T log r over the sum of T squared.

So, this would be another way of estimating the mean time to failure. And we can solve this with the numbers that we have and it is not too bad the mean is 700, so, very much within the estimate that we did from the independent time between failures approach.