

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –143
Component Reliability - Time Defined (Part - 22)

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TTF statistics - estimations from test data

Structural Reliability
Lecture 17
Component reliability
- time defined

Example : censored reliability tests

The TTF on an item is known to follow the exponential distribution. A test of 9 samples was designed to run until 3 items failed. The 3 failure instances were recorded as 144 hr, 182 hr and 243 hr, respectively. Estimate the mean time to failure of the item.

There are n samples with TTF $X_i, i = 1, \dots, n$
 X_i are IID exponential RVs with parameter λ .
 All n are put to test at time $t = 0$
 r items fail at times $\tau_1, \tau_2, \dots, \tau_r$.


ΔT_1 = the time to the first failure
 $\Delta T_1 \sim$ exponential with parameter $n\lambda$
 ΔT_2 = the time between the second and the first failure
 $\Delta T_2 \sim$ exponential with parameter $(n-1)\lambda$

Question:
Are ΔT_1 and ΔT_2 independent?
Consider:
 $P[\Delta T_2 > \Delta \tau_2 | \Delta T_1 = \tau_1]$

$$= P\left[\prod_{i=(1, n)} X_i > X_{(1)} + \Delta \tau_2 \mid \prod_{i=(1, n)} X_i > X_{(1)}, \Delta T_1 = \tau_1\right]$$

$$= P\left[\prod_{i=(1, n)} X_i > X_{(1)} + \Delta \tau_2 \mid \prod_{i=(1, n)} X_i > X_{(1)}, X_{(1)} = \tau_1\right]$$

$$= P\left[\prod_{i=(1, n)} X_i > \tau_1 + \Delta \tau_2 \mid \prod_{i=(1, n)} X_i > \tau_1\right]$$



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So we are continuing with this example on censored reliability test which is designed to stop as soon as a fixed number of samples fail. So, here we have an example of a test starting with nine samples and stopped as soon as three failures were recorded and the time to failure for each of them was recorded and we need to estimate the mean time to failure. What we have done already is we have derived the distribution for the time to the first failure that is exponential with mean with parameter and lambda.

We have found the distribution of delta T 2 the time between the second and the first failure that is also exponential with a parameter n - 1 lambda at this point we want to ask the question are delta T 1 and delta 2 to independent it would be good to know because such information helps in the estimation process. So, we start with the basics we start with how we derived the CDF of delta T 2. So, now we are conditioning it on a fixed value of delta T 1 and if we can show that that conditioning does not matter the two events are independent then we could conclude that the

random variables are independent.

So, let us proceed step by step uh. So, we start with the definition of delta T 2 in terms of all the individual axes as we had done before remember that that includes the time to the first failure x subscript parenthesis one and in the conditioning event we now have delta T 1 is equal to tau 1. So, that is what we said in the previous line. Now we can proceed we because delta t1 equals star one essentially means that the time to the first failure is tau one.

So, if we agree with that then we can replace x subscript parenthesis 1 the time to the first failure with the conditioning event that it is equal to tau 1. So, our probability our statement simplifies to the fact that all the x's are greater than tau 1 plus delta tau 2 except the one that failed already given that all the others are greater than tau one except the one that has failed at tau 1.

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<p>There are n samples with TTF $X_i, i = 1, \dots, n$</p> <p>X_i are IID exponential RVs with parameter λ.</p> <p>All n are put to test at time $t = 0$</p> <p>r items fail at times $\tau_1, \tau_2, \dots, \tau_r$.</p> <p>$\Delta T_1$ = the time to the first failure</p> <p>ΔT_1 ~ exponential with parameter $n\lambda$</p> <p>ΔT_2 = the time between the second and the first failure</p> <p>ΔT_2 ~ exponential with parameter $(n-1)\lambda$</p>	$P(\Delta T_2 > \Delta \tau_2 \Delta T_1 = \tau_1)$ $= P\left(\prod_{i=1, i \neq 1}^n X_i > \tau_1 + \Delta \tau_2\right)$ $= P\left(\prod_{i=1, i \neq 1}^n X_i > \tau_1\right)$ $= \frac{\exp(-(n-1)\lambda(\tau_1 + \Delta \tau_2))}{\exp(-(n-1)\lambda\tau_1)}$ $= \exp(-(n-1)\lambda\Delta \tau_2)$ $= P(\Delta T_2 > \Delta \tau_2)$
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Now let us proceed with the algebra we express that P a given b in terms of P ab over p b because ab one already contains in the other. So, the numerator is simpler it begins to look familiar now and if we just plug in the exponential nature of all the x's we end up with two exponential functions and when we subtract one exponent from the other we are left with the conditional probability of delta T 2 given delta T 1 has a particular value we get back the unconditional probabilities.

So, and this is since this tau 1 was arbitrary we can conclude that delta T 1 and delta T 2 are independent.

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 $\Delta T_2 \sim$ exponential with parameter $(n-1)\lambda$

Proceeding this way, we can show
 the time between j^{th} and $(j-1)^{\text{th}}$ failures:
 $\Delta T_j \sim$ exponential with parameter $(n-j+1)\lambda$
 and $\{\Delta T_j\}$ are mutually independent

Thus:
 $\Delta T_1, \Delta T_2, \dots, \Delta T_r$ are exponentially distributed
 They are mutually independent
 But not identically distributed



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So, proceeding this way we can show that the time between any two successive failures delta t_j will be exponential with parameter n - j + 1 times lambda and all the T_js all the delta T_js are mutually independent. So, we have come to a good point in this in this derivation in solving this problem is that the times between the successive failures they are exponential they are mutually independent but they are not identically distributed.

So, that is one problem which still remains because you see all the parameters they are not the same it starts with n lambda then n minus 1 lambda etc. So, n - j + 1 lambda for the general case, so, they are not identically distributed.