Structural Reliability
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Lecture –142
Component Reliability - Time Defined (Part - 21)

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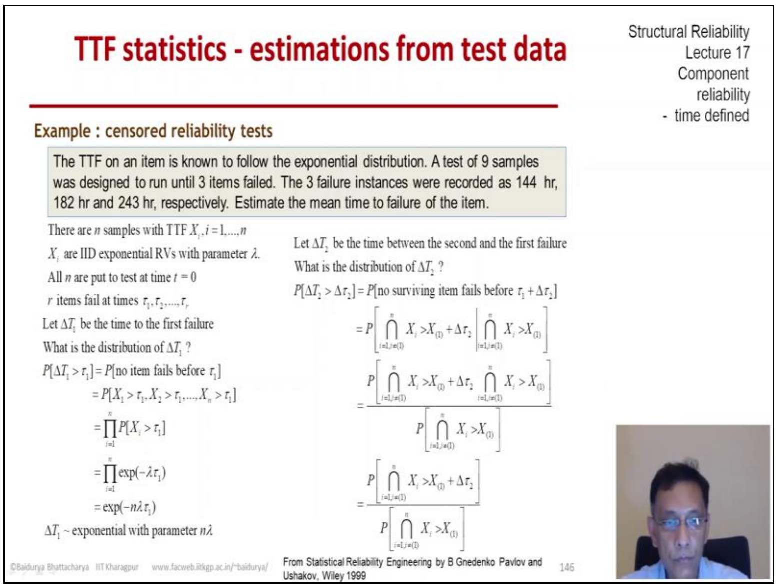

In this sensor reliability test; notice that the test stops as soon as a predetermined number of samples fails. So, in the earlier example taken from Negansko's book we stopped at a fixed time and regardless of the number of failures now we are stopping as soon as three failures occur. So, in general R failures and we record the individual failure times we need to estimate the mean time to failure and as before we know that the distribution of time to failure for the item is exponential in nature.

So, let us set the problem up carefully. So, we start with n samples each with time to failure x i the x i are IID independent and identically distributed exponential random variables with the same parameter lambda. So, I goes from 1 to n all these n samples are put to test at time t equals 0 and r items fail at time star 1 tau 2 up to tau r. So, these are the instances that we record and then we stop the test.

So, we need to estimate the mean time to failure obviously if we just take the mean of these three numbers 144, 182 and 243 it would be overly pessimistic we understand that because it is only those three that failed and six of them did not fail. So, we need to somehow take those into account. So, one way to proceed would be to define the distribution of successive times to failure. So, the time to the first failure the time to the second failure and so on up to the time to failure number R.

And we have looked at this sort of problem before when we looked at the Poisson process and the sum of individual exponentials leading to shock number k and gamma distribution for the time to kth shock and etcetera. So, this will be a similar setup but let us proceed step by step so let delta t 1 be the time to the first failure. So, how do we derive the distribution for this delta t1 this random time to first failure turns out that we will make use of the fact that all the x's are IID exponentials.

So, we do not find the CDF but we define we start with the complementary function the reliability function if you will and we are trying to find the probability here that the first time to failure that the time to the first failure is greater than some constant tau 1. So, that actually gives us a very quick answer it basically says that no item fails before tau one we are starting from zero first failure does not occur up to tau 1.

So, that is what we are asking and. So, that basically means in terms of all the x's is x 1 greater than tau 1 x 2 greater than tau 1 and all of them greater than tau 1 and because they are independent we can multiply the probabilities and because they are exponential the product of all these exponential functions turn out to be exponential of minus n lambda tau 1 because lambda is constant for all of them.

And which basically means if the complemented CDF of a random variable looks like this exponential of minus a constant times the value of the random variable we looked at it basically means that the random variable is exponential. So, delta t 1 is exponentially distributed with parameter and lambda. So, that is very good to know. So, then let us see if we can be more ambitious and try to find the distribution of the next time between values.

So, delta t 2 so we are not looking at time to failure but time between failures, so, the time delta t 2 is the time that it takes for the second failure after the first one. So, what is the distribution of this delta t 2 let us proceed similarly. So, let us take the accidents type event. So, delta t 2 greater than delta tau 2. So, that is the probability which we are trying to find now that is we have to be very careful in this definition it is basically asking for the probability of an event which is no surviving item fails before tau 1 plus delta tau 2.

That is very important no surviving item fails before tau one plus delta tau 2 the because all these elements are starting service at t equals 0 delta t 2 the time between the second and the first phase to be defined in terms of the excess. So, that is how we are doing it and that in terms of all the x's looks like this. So, each x i is greater than the time to first failure plus delta tau 2 okay. So, that is clear except we need to define that x subscript parenthesis one. So, x subscript parenthesis one is what you see on your screen that is the time to the first failure.

So, the; weakest element if you will the time that it failed. So, the first element to fail is x subscript one and. So, it's time to failure. So, this intersection for this intersection event goes for all eyes one to n except the index corresponding to the element that failed first now the surviving item. So, we this is not the end of it this is conditioned on the event that all the elements except that x subscript parenthesis one they survived.

So, they are all greater than x subscript parenthesis one. So, they are all greater than the element that failed first. So, with that now we have an event like we have a probability of a given b. So, now let us use that definition. So, P of a given b is P of ab over P b. So, that lets us just express the same probability with in terms of a fraction. So, now let us look at the numerator the numerator the intersection of those 2 events just clear that the one on the right is already contained in the one in the left.

So, the intersection of the 2 would be just the left one. So, we can simplify the numerator as intersection of all the x's greater than the time to the first failure the random time to the first failure plus that delta tau 2 that we have that we started with and in the new in the denominator we have all the surviving elements greater than other time to the first failure. So, that is the point that we at which we then bring in another conditioning.

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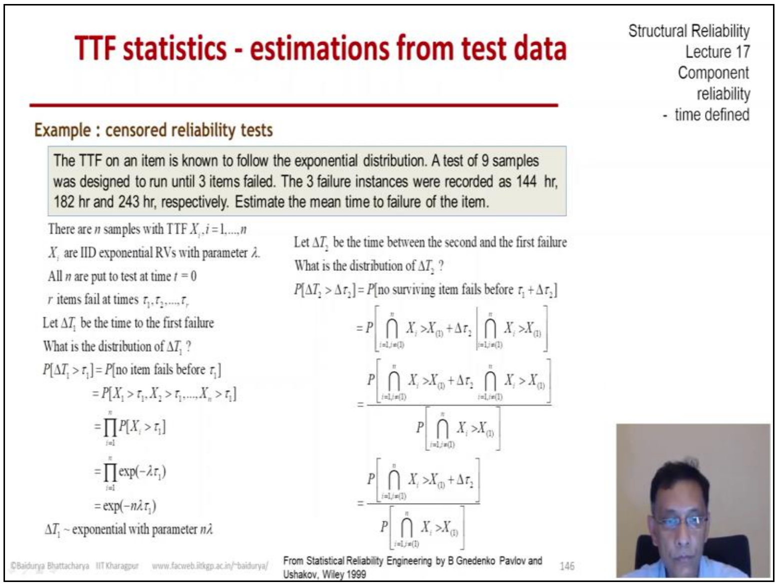

So, let us move to the next slide uh. So, we have already found out that delta t 1 is exponential with parameter n lambda and now we are in the middle of finding the distribution for delta t 2 the time between the second and the first failure and we ended with a ratio of 2 probabilities. Now let us condition both the numerator and the denominator on the time to first failure. So, x subscript parenthesis one, so, we are going to use the theorem of total probability on x one taking all possible values tau one integrated over the entire range. So, we have done this sort of formulation before many times.

So, let us just do it rightly step by step and then once this is done we can introduce we can enter the density functions the distribution of all the x's so and we also use the fact that all the x's are independent and identically distributed. So, given that x subscript parenthesis 1 is tau 1 that does not change the distribution of the individual x's all the other x's. So, we can invoke the exponential nature of all of them there are n - 1 of them.

So, we multiply all of them in the numerator and also in the denominator once we do that we can factor out the term that is independent of tau 1. So, we bring it out of the integration in the numerator and what we end up with are 2 common factors in the numerator and the denominator and they can they cancel out each other and what we are left with is a simple expression of exponential of negative a constant times delta tau 2 which is what we started with.

So, as soon as a random variable has a complementary CDF looking like that it must be an exponential random variable. So, just like delta t 1 delta t 2 is also an exponential random variable with parameter n - 1 lambda delta t 1 was had parameter n lambda delta t 2 has parameter n - 1 lambda.