Structural Reliability
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Lecture –141
Component Reliability - Time Defined (Part - 20)

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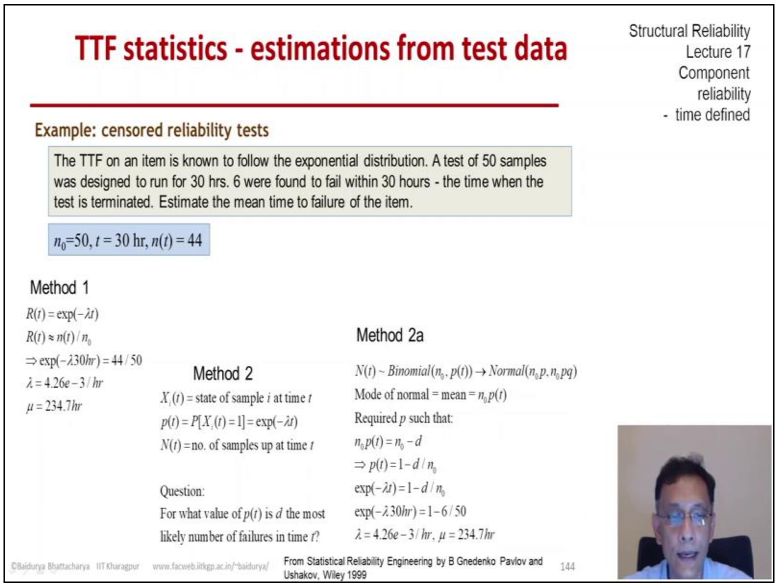

For the remaining examples of this lecture we are going to look at a couple of examples on censored tests. And these examples are taken from the book by Nyadenko Pavlov and Ushakov. So, when we talk about a sensor test a very common way to do that would be to have a predetermined duration of the test. So, we will not wait for all the items to fail if that predetermined duration is exceeded we will stop the test.

So, that is one kind the other common kind would be to fix a number of failures that we will wait for. So, as soon as that number is reached we will stop the test. So, we will not continue until all the samples have failed. So, here let us take a minute to read the problem we have we have 50 samples of a test that is designed to run for 30 hours and within those 30 hours 6 were found to fail and then the test stopped.

So, now we need to estimate the mean time to failure now here is a very important difference from what we did before it is given that the time to failure follows the exponential distribution. So, the examples we looked at before were non-parametric but here we have knowledge about the distribution of the time to failure which is defined by one parameter lambda. So, now we are entering the domain of parametric tests also our parametric estimations.

Also we are not going to go deep into theory of estimation we are just going to learn the basics how to use this. So, all our estimates would be point estimates it will we will not have interval estimates or test of hypothesis and so on. So, we will just concentrate on the data and how to make some good inferences from the data that is that is our objective. So, here we have an exponential time to failure and we all we know is that 6 failed within 30 hours out of 50.
We do not know when they failed and we don't know how it would continue beyond 30 hours because the test was stopped.

So, what options do we have there are two ways that we could proceed. So, n 0 is 50 t is 30 hours and n of t is 44 that much we know and that is the only thing we know except beyond that the time to failure is exponential. So, we can write the reliability function which is exponential minus lambda t lambda is the parameter of the distribution and we can also estimate the reliability as the fraction surviving.

So, n t over n 0 now we could relate the two the theoretical value of the reliability and the observed value of reliability and again just a point estimate not going through any more sophisticated methods of likelihood or interval estimates we will just do a point estimate equate the 2 and it turns out that lambda from this condition gives me 4.26 10 to the minus 3 per hour and the reciprocal is the mean.

So, that is about 235 hours that would be a very quick and efficient way of solving the problem we could also ask a different sort of question and that would be that let us say that each item behaves as a binary element evaluated at time t. So, the state of the sample at time t if we call that x i then it could either be up or it could be down. So, the probability of being up is that it survives up to time t.

So, that we already know is exponential minus lambda t because the time to failure is exponentially distributed. So we could define a new random variable capital N of t which is the number of samples that are up at time t. Now if these samples are independent, we have actually solved a single example in the past. So, this is actually a binomial random variable this capital L of t and the question is for what value of p is d the most likely number of failures to occur within time t.

So, let us see if we can answer that question in a reasonable manner and what answer it would lead to and we will do it in two ways. So, method two a is as I said n t is binomial with two parameters n 0 and p of t. Now we know that for reasonably large N and a finite n p the binomial approaches a normal distribution. So, the normal mean is n 0 p and the normal variance is n 0 pq. So, because the normal is that the mean is equal to the mode.

So, the mode is the most likely value. So, basically we are asking the question what makes the observed number of survivals the mean or the mode of this distribution because we have defined p as the survival probability. So our equivalent statement is that what value of p gives me the number of survivals most likely to be 44 because that is what I have observed which means what value of lambda would give me that that is the that is the meaning actually.

So, that gives me p of t would be 1 - d over n 0 just rearranging and that using the expression for p of t I get an x I get a relation between lambda and and d and which if I do the algebra i end up with actually the same answer as I had in method 1 is lambda is 4.26 10 to the -3 per hour and the mean is 234.7 hours. So, whether we equate the reliability or whether we say that what is the most; what is the value of d that would make; what is the value of lambda that would make d the most likely value observed of failures gives me the same answer.

I could not make the normal approximation I could stay with binomial and let us see what answer we get in that case.

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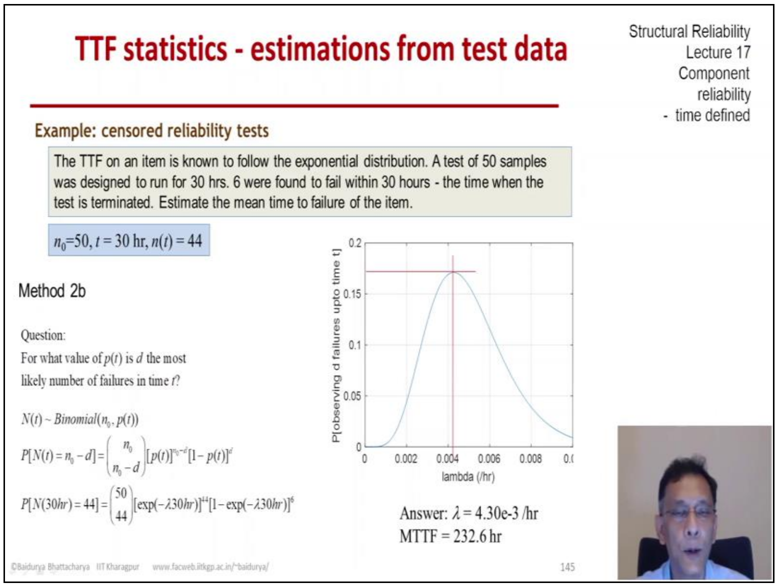

So, the question is that what value of p makes d the most likely observed number of failures h. So, n is binomial as i said and i'll not make the approximation. So, this is the binomial probability of observing d failures or n 0 - d successes. So, there are 44 or n 0 - d items that are up at time t. So, the probability of that is dependent on lambda clearly as we see. Now if I just plot the value of this probability n equals 44 as a function of lambda that is the graph that you see on the right.

And clearly it is there is one peak and if I find out the value of lambda that gives me that peak it turns out to be about four point three ten to the minus three per hour very close to what we had before a little different and the mean is coming to 232.6 instead of 234.7. So, this is another way of estimating the mean time to failure from a censored test with a known distribution for the time to failure.