Structural Reliability
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Lecture –139
Component Reliability - Time Defined (Part - 18)

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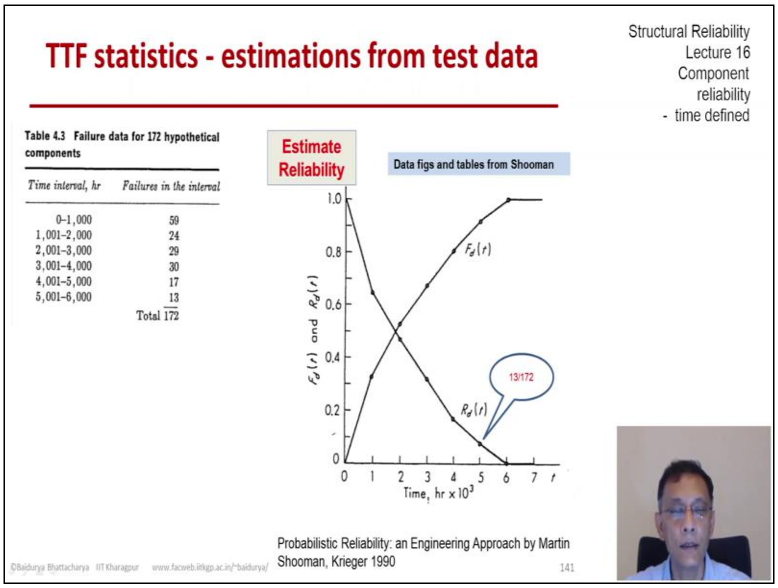

The next example we take up is also from Schumann's book we have here failure data presented in intervals of time. So as you see on the table on the left there were 59 failures between 0 to 1000 hours of test time and then for subsequent thousand hour intervals all the way up to six thousand. So, we have 6 intervals and a total number of 172 items tested. So, the first task will be to estimate the CDF of this item.

So, after of the time to failure of this item, so we let us reproduce the figure from Truman's book and we will make sure that we are able to reproduce the results. So, let us take it interval by interval. So, obviously when time equals 0 the CDF is 0. So, that is that is the origin from which we will start. So, at the end of 1000 hours 59 failures have occurred. So, that gives us one point.

So, on the time axis 1000 hours and on the CDF axis we have the next point which is 59 over 172. So, that is the definition of the CDF according to our estimation strategy. So, the first point at the end of the first interval gives us 59 over 172 and let us continue. So, the next one are 24 additional failures occurred. So, 59 plus 24 is 83 divided by 172 is the next data point on the CDF ax curve the next data point is another 29. So, that gives us 112.

The next data point is another 30 failures. So, that is 142 that is the next point on the CDF curve and 17 more failures. So, that is 159 over 172 and finally when all items have failed that's the end point we are taking at 6 000 because that is the data that is presented we do not know exactly when the last failure occurred. So, at 6000 the CDF comes to one. So, that is how we are able to estimate CDF from data presented in terms of intervals.

We could do the same thing for the reliability function and we well just do the complement of that what we did for the CDF. So, at t equals 0 we would trace the reliability curve that you already see at t equals 0 it is 1. So, we start with reliability of one and then we could come down to the next one. So, at 1 113 are surviving. So, now we are looking at the survival fraction. So, that is 113 over 172 and so on.

We do not need to trace all the points we get the idea. So, we can at the end of 4 000 hours there are 30 items surviving. So, that's 30 over 172 at the end of 5000 hours 13 items are surviving as we see from the table. So, that is the reliability 13 over 172 and at the end of 6000 hours they have all failed. So, the reliability comes down to 0.

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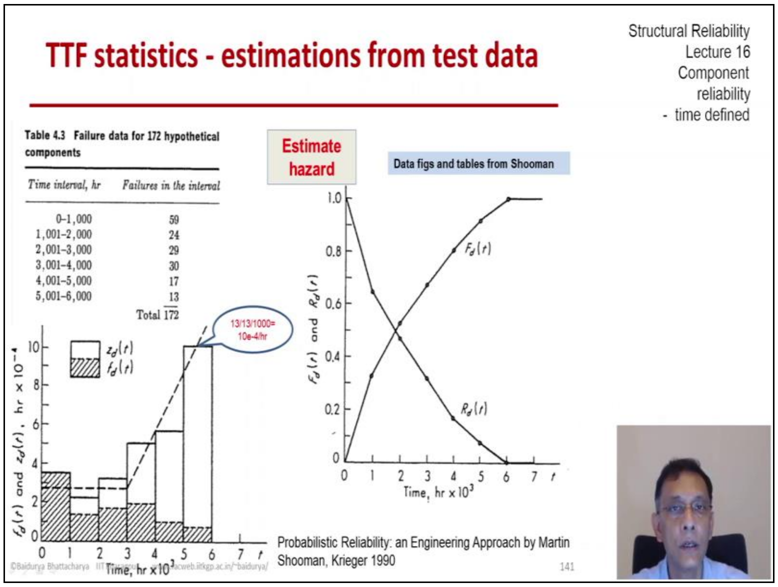

The next task is to estimate the PDF of the time to failure of this component from the same data that is in our table. So, for that let us reproduce the graph from Schumann's book and this particular graph includes both the PDF and the hazard function and according to the author's nomenclature z d stands for the hazard function and f t small f t stands for the density function. So, let us now make sure that we are able to reproduce the PDF values from this data set.

So, the first one is let us apply the formula carefully. So, 59 failures happened in the first interval of 1000 hours out of 172. So, we are basically differentiating the CDF numerically. So, that is 59 over 172 over 1000 hours. So, that is about 3.4 10 to the -4 per hour. So, that is the PDF corresponding to the first interval. The second intervals there are 24 failures and proceeding the same way 24 over the original number of samples over the duration of the interval. So, that gives me 1.4 10 to the -4 per hour.

The next one is 29 failures occurred in the interval 2000 to 3000 hours and that gives me 1.7 10 -4 per hour the next one there were 30 failures between 3 and 4 000 hours. So, that gives me 1.7 10 to minus 4 per hour. The next one is 17 failures occurred note in each case the new the denominator is constant because the first number in the denominator is the n 0. So, that obviously does not change the second number in the denominator 1000 that is the length of the time interval. So, these are equal intervals.

So, that number turns out to remain constant in this example. And the last is 13 failures occurred between 5 and 6000 hours. So, that gives me 0.7 into 10 to the -4 per hour. Let us now estimate the hazard function. So, the hazard function looks at the ratio of the PDF and the reliability function that's one way of estimating it. So, here. So, then we have 59 failures 172 items at the beginning of the interval that number in the denominator is not going to stay constant when we do the hazard function unlike the density function but because we are starting from the initial condition.

So, the hazard and the PDF are equal for the first interval the second interval we have 24 failures but we started with 113. So, that divided by the length of the interval gives us 2.1 10-4 per hour and so on. So, we could continue with this process number of failures over the number of surviving elements to start with and the length of the interval. So, we could keep doing that until we reach the last interval in which we started with 13, 13 failed in 1000 hours. So, that's 10 to the -3 per hour.

So, that would be how we estimate the hazard function for data presented in a tabular form and clearly it seems that there is something going on about aging some sort of damage seems to be accumulating in the component because the hazard function clearly shoots up beyond about 3000 hours or so. So this is a case quite possibly where the exponential time to failure description is not going to be valid this is not a case of a constant failure rate but this is a case of increasing failure at least beyond about 3000 hours of service life.