Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –138
Component Reliability - Time Defined (Part - 17)

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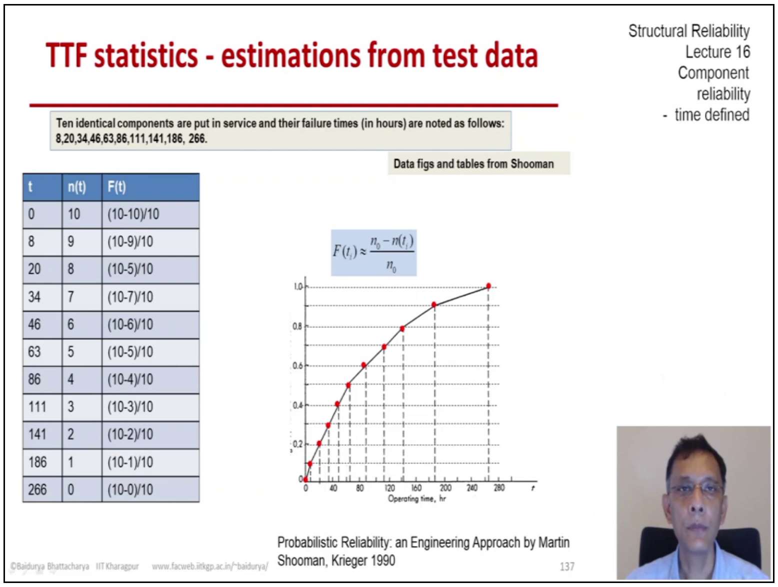

In this and the next three slides we are going to analyze an example taken from Schumann's book as you see on the screen there are 10 identical components that were put in service and they all failed one after the other and those 10 failure times were noted. And so they are starting from 8 hours all the way up to 266 hours there are 10 of them. So, we will use this very simple data set to find the CDF of the time to failure of that component.

The PDF the reliability function and the hazard function. So let us take this up one by one. So, let us first list the data in a table. So, we have t starting from 0 all the way up to the last failure time 266 hours and the number of surviving elements dropped from 10 all the way up to 0. So, the number of surviving elements is 10 at t equals 0 obviously and there is no surviving element at time 266.

So, first we are going to estimate the CDF of t capital F. So, this is how we are going to use the estimation. So, it is the number of failed units and 0 minus n t over n 0. So, let us take it up one by one. So, what I have done I have also taken the graph from Schumann's book for this problem. So, but you see the CDF curve there but we are going to derive this point by point. So, that we understand the process uh. So, at t equals 0 there is no failed element.

So, CDF is 10 minus 10 over 10 so the value is 0 and that is the red point that you see. The next is at 8 hours of time. So, we have one failure. So, the at 8 hours we have the next discrete point on the CDF curve identified and that is the next red dot that you see at the next failure time 20 hours we have. The next start so, that is 10 - 8 over 10 and there is a mistyping there it is 10 -8. Anyway, let us continue at 34 hours we have 10 - 7 over 10.

And that is the next red dot that we have plotted and so on. So, if we keep doing this we are going to trace the curve that is already there from Schumann's book or more correctly we are going to get those red points. And then adjoin them with a with the best fit curve but this would be the process of point wise finding the estimated CDF the x-axis we have the observed failure times and this is there is one failure at each that we have observed there could be other databases in which we could give number of failures in a certain interval of time.

So, we are going to see an example of that later on as well. So, here we have the 11 points giving me the 11 estimates of the CDF of the time to failure going all the way from 0 to 1.

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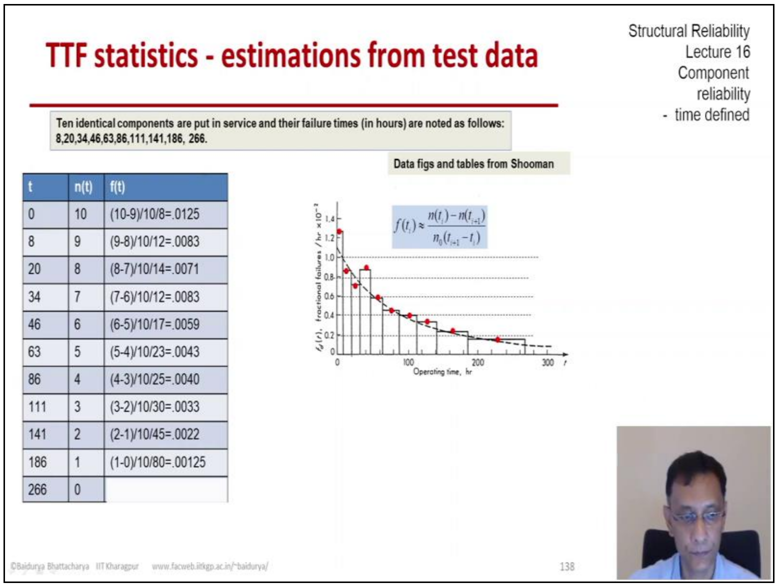

We continue with the same data set and this time we estimate the PDF the density function of the time to failure. So, we have the same table t going from 0 to 266 hours and the number of surviving elements going down from 10 to 0 and we are going to estimate the PDF with our formula that we had put up a few slides ago. So, it is the increment in the number of failures or the drop in the number of survivals over the original number of components and the time interval over which that happened.

So, basically we are differentiating CDF uh. So, let us let us do it point by point now obviously since we have a t i and the next step t i + 1 we are not going to have 11 estimates as we did for the CDF because now we are dealing with the derivative and we are going to only have 10 estimates of the of the PDF. So, the first red point is for the interval 0 to 10 and we use the formula 10 - 9 divided by 10 and the duration over which that happened 8 hours.

So, that gives me 0.0125 per hour and then the next one um it's the the duration is now 12 hours and the that interval is identified by the second red dot that you see this way we move on to the next one which is a duration of 14 hours and that gives me 0.0071 per hour and that is the third red dot and so on until we can exhaust all our 10 intervals. And we can see a trend here definitely it is a drop it could be an exponential drop.

We do not know we need further tests to determine that but one good verification of whether it is an exponential time to failure or not could be you know guessed if if the hazard function is kind of a straight line if we see a more or less flat hazard curve for the entire range of observed times then we would be more convinced that this is probably coming from an exponentially distributed time to failure.

So, we proceed the last interval has a duration of 80 hours and one failure obviously. So, that ends the PDF with 0.00125 per hour.

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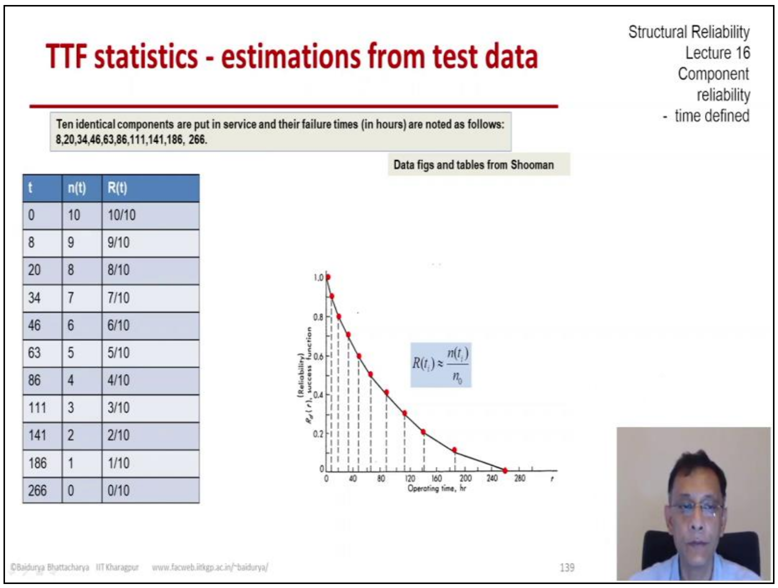

The next function to look at for the same data set is the reliability function. So, here is the table once more we have the times observed at each successive failures t going from 0 to 266 hours and number of surviving elements are dropping from 10 to 1 10 to 0 and we would like to use the expression that you see on the on the screen to estimate reliability. So, its number of surviving elements at time t over the original number of elements n0 this is the graph reproduced from Truman's book and let us make sure that we can reproduce it for from the data.

So, the first one is at time 0 reliability is one. So, it is 10 surviving over 10 original and then the next one is 9 over 10 and so on. So, there is no surprise there we have done the complement already with the CDF and. So, we climb down from one all the way to 0 going through each observation of failure and then once we are done with these 11 points we could join them with piecewise continuous lines or a smooth curve and estimate the reliability function the area under this curve would be as we have derived before the mean time to failure.

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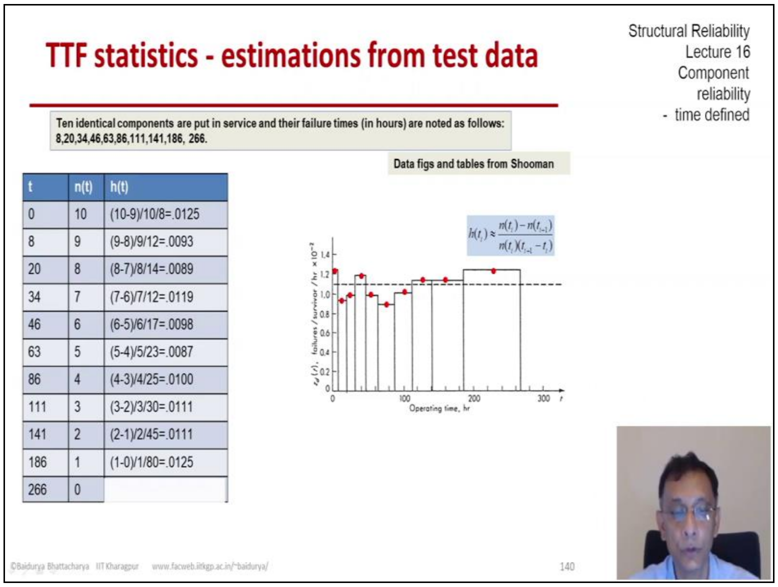

The last function we estimate from this data set is the hazard function. So, let us put the table on the screen first. So, we have the observed times to failure the number of surviving elements and then we are going to use the formula n t - n t + n t i – n t i + 1. So, again we are estimating with one step forward. So, we will be able to have only 10 estimates and then that divided by the number of surviving elements at a at the time in instant of interest and divide that by the length of the interval.

So, if you remember we are dividing the density function with the reliability function so let us do that step by step this is the result taken from Schumann's book. So, but make sure let us make sure that we get the same answers. So, the first one is one failure. So, n t i - n t i + 1. So, delta n is one which is the same for all these observations all these intervals that we have but the number of the length of the time interval is is not the same.

So, that is going to create some difference. So, we have 10 minus 9 divided by 10. So, which is the current number of surviving elements and that divided by the length of the interval gives me 0.0125 per hour that is the the hazard corresponding to the first interval from 0 to 8 hours for the next interval 8 to 20 hours we have one failure with the initial number of failures initial surviving numbers being 9 and the length of the interval 8 to 20 is 12 hours.

So, that gives me 0.0093 per hour the third one again has an interval of 14 hours and eight items to start with. So, that gives me 0.0089 and so on. So, we continue down the list and we can trace all the numbers and more or less i think the answers match perfectly and. So, we come to the interval number eight and then number 9 and then 10. So, what we see here as we had suspected when we are looking at the PDF is that it kind of looks like an exponential drop.

And here we have a hazard function that one could argue very convincingly that it is more or less a flat constant line a horizontal line. So, there is reason to believe that there are good reasons to believe that this time to failure is exponentially distributed and within noise within observational error we have observed a more or less flat hazard curve. So, this would be the way that we would analyze a simple data set of observed failure times.

When we do not have any parametric tests in mind we are not fitting to any particular distribution we do not have any underlying assumptions all we have is the is the data and we are estimating the hazard function the reliability function and the mean time to failure if necessary from the data itself without bringing in any additional assumptions.