

**Structural Reliability**  
**Prof. Baidurya Bhattacharya**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture –137**  
**Component Reliability - Time Defined (Part - 16)**

We continue with our discussion on time to failure based approach to reliability. Today we are going to talk about estimation of time to failure statistics from test data.

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**Structural reliability - course recap**

- Part A
  - Motivation
  - Basics of probability
  - Basics of random variables
  - Common probability distributions
  - Joint distributions
  - Monte Carlo simulations - discrete continuous and dependent variate generation
- Part B
  - History and scope of reliability studies
  - Definition and terminologies
  - Reliability problem formulation
  - System representation & redundancy
  - Time to failure based approach to reliability
  - Random TTF, MTTF, hazard function
  - Estimation of TTF statistics from test data

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Let us recap what we have done in the course. So, far we completed part a in which we talked about the basics of probability basics of random variables, common discrete and continuous probability distributions joint distributions and then Monte Carlo simulations and generation of discrete and continuous and dependent variables. Part B which is going on now started with the history and scope of reliability studies and then we defined all the relevant terms we discussed a lot on how to formulate problems in reliability because that is one of the most fundamental steps.

We discussed how to represent systems in terms of its elements and we ended that discussion with different approaches to redundancy then we started time to failure based approach to reliability in a phenomenological way. So, what we have is a random time to failure TTF and we describe reliability and related matrix based on the time to failure properties. So, we have mean

time to failure we have hazard function and so on.

So, today we are going to talk about how given test data on time to failure. We can estimate the reliability function the hazard function the mean time to failure and. So, on we will do that both for non-parametric and parametric cases and we are going to end this part with a discussion on system reliability from a time to failure based approach.

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## TTF statistics - estimations from test data

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Recall:

In a test program involving identical specimens subject to same conditions:  
 $n(t)$  = no. of specimens surviving at time  $t$ ,  
 $n_0$  = no. of specimens at the start of test


$T$  = random time to failure (TTF)  
 $F(t) = P\{T \leq t\}$   
 $f(t) = \frac{d}{dt}F(t)$   
 $R(t) = 1 - F(t)$   
 $h(t) = \frac{f(t)}{R(t)}$

$F(t) \approx \frac{n_0 - n(t)}{n_0}$

$f(t) \approx \frac{n(t) - n(t_{i+1})}{n_0(t_{i+1} - t)}$

$R(t) \approx \frac{n(t)}{n_0}$

$h(t) \approx \frac{n(t) - n(t_{i+1})}{n(t)(t_{i+1} - t)}$



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So, let us let us recap the test setup that we had mentioned before we start with  $n_0$  specimens and then at any given time  $t$  in the future after the test starts  $n(t)$  specimens remains. So,  $n(t)$  is the number of surviving elements at time  $t$  and just based on this we can come up with estimates of all the functions related to time to failure. So, if capital  $T$  is my random time to failure TTF then the CDF is the probability that capital  $t$  is less than small  $t$  less than or equal to small  $t$  the density function the PDF small  $F$ .

That is the derivative of the CDF capital  $F$  the reliability function is 1 minus the CDF capital  $F$  and the hazard function which is the last item that we learned is the ratio of the density function with the reliability function. So, we had estimates for all of them the CDF at any time  $t_i$  can be estimated by the number of failed samples over the original number of samples the density function can be deduced from the increment of the CDF over the time over which it happened.

So,  $F$  of  $t_i$  plus  $1 - f$  of  $t_i$  capital  $F$  divided by the  $\Delta t$  would give me an estimate of the density function small  $f$  the complement of the CDF gives me the reliability function. So, the fractional number of items that are surviving at time  $t_i$ . And then finally the hazard function which is the ratio of the density function over the cumulative distribution function over the reliability function and that is the increment in  $n$  between  $t_i$  and  $t_{i+1}$  and normalized by the number of surviving elements at  $t_i$  over the time interval.

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$T$  = random time to failure (TTF)  
 $F(t) = P\{T \leq t\}$   
 $f(t) = \frac{d}{dt}F(t)$   
 $R(t) = 1 - F(t)$   
 $h(t) = \frac{f(t)}{R(t)}$

$$R(t) = \exp\left[-\int_0^t h(x)dx\right]$$

$$= \exp\left[-\int_0^{t_1} h(\tau)d\tau\right] \exp\left[-\int_{t_1}^{t_2} h(\tau)d\tau\right] \dots \exp\left[-\int_{t_{n-1}}^t h(\tau)d\tau\right]$$

$$\approx (1 - h(t_1)\Delta t_1)(1 - h(t_2)\Delta t_2) \dots (1 - h(t_n)\Delta t_n)$$

$R(t) \approx \prod_{i=1}^n (1 - h(t_i)\Delta t_i)$

$$f(t) = h(t)R(t) = h(t) \exp\left[-\int_0^t h(\tau)d\tau\right]$$

$f(t) \approx h(t) \prod_{i=1}^n (1 - h(t_i)\Delta t_i)$

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So, this is what we learned and then there are certain other equivalent ways of this estimating these quantities for example we could start with the relationship between reliability function and the hazard function as you remember  $R$  of  $t$  is exponential of negative the integral of  $h$  of  $x$  between  $0$  and  $t$ ,  $h$  being the hazard function and now that we could split into a number of intervals. So, that  $0$  to  $t$  we could split into  $0$  to  $t_1$  and then  $t_1$  to  $t_2$  and all the way up to  $t_{n-1}$  to  $t_n$ .

And each of these exponentials we can approximate to the first linear term. So, the first term becomes  $1 - h$  of  $t_1$  times the length of the interval from  $0$  to  $t_1$  the next one is  $1 - h$  of  $t_2$  times  $t_2 - t_1$  and so on. So, we have a product of all these  $1 - h \Delta t$  and that would also give me an approximation of the reliability function. So, that is what you see on the screen, based on that we can estimate the density function.

So, the density function by definition is the hazard times reliability function and we could take the same approach as in the first half of that slide and we can express the density function in terms of the hazard function times the product that you saw on the first part. So, these are the various ways in which we can estimate all these functions related to time to failure from test data.