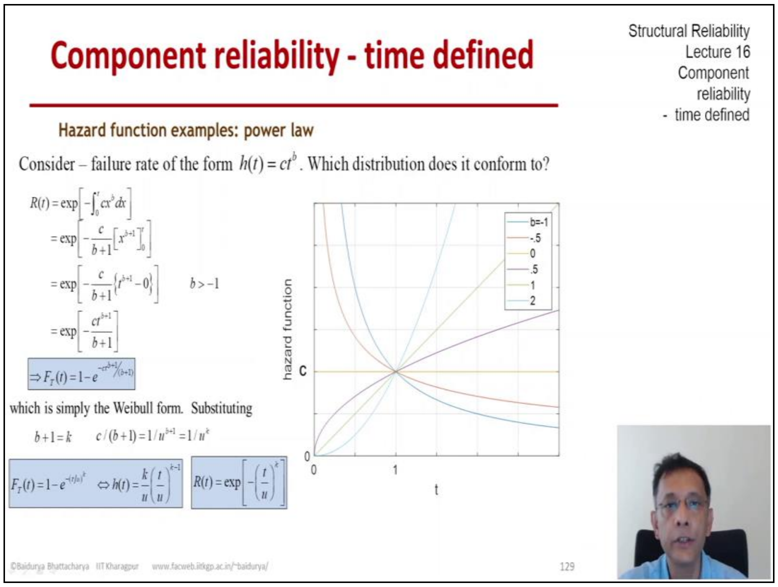
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –134
Component Reliability - Time Defined (Part - 13)

(Refer Slide Time: 00:24)



We now look at a very simple form of the hazard function a power law type but it turns out to be very versatile. So, h of t is simply c times t to the power of b where c and b are 2 constants. So, parameterized by these 2 what possible shapes can h take first and then which distribution does this hazard function conform to. So, let us let us derive that step by step. So, if we plot h versus t we see that it is capable of as a power law does reproducing all sorts of shapes. So, when b is 0 then we obviously get the constant hazard function constant failure rate.

So, immediately we see that this corresponds to the exponential time to failure now but what about the others because for b greater than 0 we have an increasing type as a function. So, you can see b, I have 3 values there b of 0.5 b of 1 and b of 2. And you can see that they are all of an increasing nature and rising faster and faster obviously as b increases on the other hand when b is less than 0. So, b is say minus 0.5 negative 0.5.

Then we have the orange curve which gracefully starts from a very high value and then keeps falling and falling, b equals -1 is also shown there that has even a faster draw but obviously we have to be careful is this even a legitimate hazard function we have to worry about that but. So, when b is equal to 0 we know that it's acceptable because we've already seen that it comes from the exponential but for other values of b do they give rise to legitimate hazard functions and if so, what are the distributions?

So, let us write out the reliability function in terms of the parallel hazard function and it is straightforward it is exponential of negative integral from 0 to t of c x to the power b dx x is the dummy variable of course. So we can go through the steps and we come up with an expression exponential of negative c time's t to the power of b + 1 divided by b + 1. And we need to make sure that this is valid when b is greater than negative one so b equals -1 is not going to be admissible.

Now let us subtract that from one and that gives me the CDF of the time to failure whose hazard function has that power load type c t to the power of b. So, this should look familiar and those of you who have gone through all the different functional forms of the CDF's you have recognized by now that this is simply the bible form and just to make sure that we put it back in the familiar form of the viable let us do some substitutions.

So, we for b and c we substitute 2 quantities k and u just to use familiar notations. So, b plus 1 we substitute with k and 1 over c over b plus 1 we substitute with 1 over u to the power of k. So, that brings us to more familiar territories and that clearly looks like that the viable form which is 1 minus exponential negative t over u whole to the power of k. So, that is what we have seen viable before and that just to make sure is the two parameter variable that we have identified in the past.

And so, that is the relationship between the hazard function form and the CDF form because we have brought everything down to u and k. So, the hazard function if you like is expressed in terms of k and u on your screen. So, t is raised to the power of k - 1 and the reliability function is now simply exponential of minus t over u to the power of k and that is what we saw. So, when k is equal to 1 b 0 k is equal to 1 that is when we get the exponential form the constant failure rate.

So, interestingly we see that the viable distribution is quite versatile it can model a constant failure rate the exponential special case and increasing failure rates and decreasing failure rates. So, we should feel very safe in choosing the viable for a time to failure random variable and once we have the data we can see if it is decreasing type or increasing type or constant and then select the parameters accordingly.