Structural Reliability
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Lecture –133
Component Reliability - Time Defined (Part - 12)

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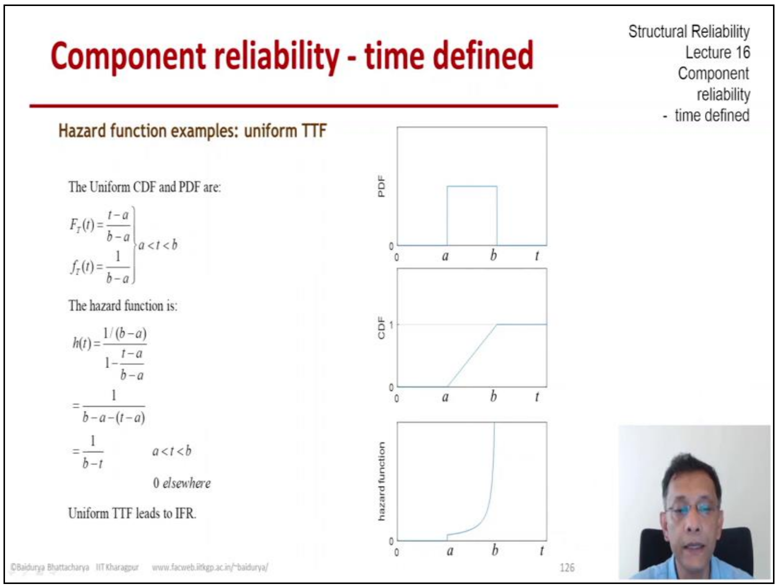

In the next few slides we are going to look at a few well-known distributions for the time to failure and investigate what sort of hazard function they give rise to we are going to pay some attention to the shape of the hazard function as well. So, let us start with the uniform distributed time to failure and as you remember the uniform random variable is defined between two constants a and b. So, what you see on the screen are the uniform CDF and the uniform PDF the PDF is a rectangle between the limits a and b.

So, the hazard function let us start from basic definitions it is the ratio of the PDF to the reliability function which is 1 - CDF and that is what you see on the screen and if we simplify it we end up at a hazard function which is equal to 1 over b – t, t being the realization of the random variable between a and b only at 0 elsewhere. So, now we can ask an interesting question is that you know we have looked at certain properties of the hazard function that its integral between 0 and infinity has to blow up it is it should be it should tend to infinity.

So, can we have that property when the hazard function is defined only in a very finite range and 0 outside of that range? It turns out that yes the random variable does not have to go all the way up to infinity have a density function defined for very, very large values in order for the hazard function to exist and for a legitimate reliability function to be defined based on that random time to failure.

So, here we have a very good example of a random variable that is defined on a finite range only both on the left and the right but still we have a reliability function which starts from 0 ends at which starts from one ends at 0 and the hazard function is non-zero only within that finite range. So, let us see what it looks like and obviously 1 - 1 over b - t is an increasing function. So, the uniform TTF leads to an increasing failure rate and increasing hazard function.

So, that actually has a significance we are going to discuss in detail what an increasing failure rate means and what a decreasing failure rate means when we discuss the buffed up curve we have already discussed the constant failure rate which comes from the exponential random variable. So and the constant failure rate basically says whatever time you are at the likelihood of failure the next instant does not change.

So, there is no aging effect going on things do not become more likely to fail as time goes for an exponential TTF. So, the constant failure rate is a very special type of hazard function but here the uniform TTF leads to an increasing failure rate and this is what the PDF looks like as i said it is a rectangle and the CDF is a linear function between a and b rises from 0 to 1 and the hazard function now goes up asymptotically to infinity at b but the area under this obviously is infinite. So, that the reliability function at b comes down to 0.

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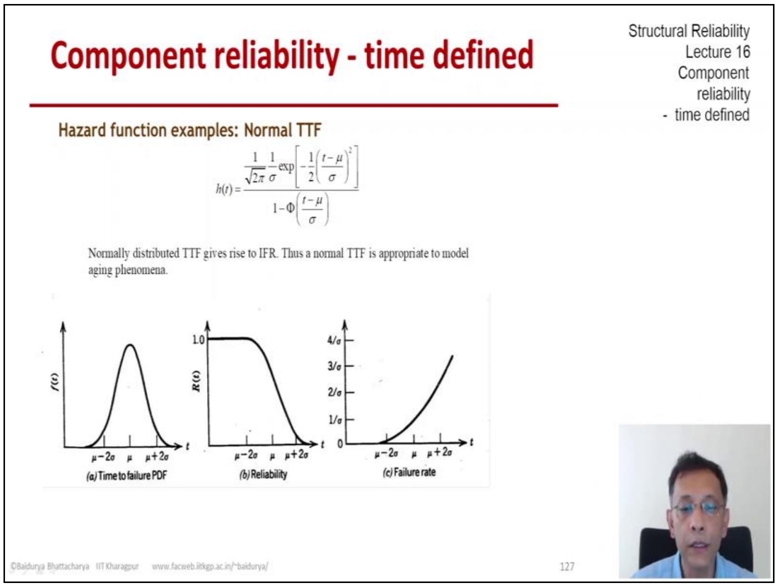

Next we look at the normally distributed time to failure obviously we have to be careful normal distribution is defined between minus and plus infinity but we are talking about the time to failure only taking non negative values. So, let us make sure that we understand this that even though we have a normally distributed TTF we are defining it for only positive values of t which means that we have truncated the distribution at t equals 0 left truncated and we have normalized the density function. So, that it stills it checks all the boxes. So, if that is.

So, then the hazard function is the ratio of the PDF and one minus the CDF and that is what we see on the screen and how does the shape of this hazard function look like and we are going to reveal that in a second. So, this is the density function centered on mu and as if mu is large enough you can see that in relationship to sigma. So, possibility of negative values is small but we can still left truncated now this is the reliability function falling from 1 to 0 and then the hazard function is actually increasing in nature.

So, for a normally distributed time to failure we have an increasing failure rate and increasing hazard function and basically we are looking at items that become more likely to failure as time goes on. So they are looking at aging type behavior and we are going to look at this in detail as I said later. So, the normal distribution gives rise to a an increasing failure rate now all the objections related to possible negative values are normal they are eliminated when we have a log normal time to failure. So, the log number distribution is defined only for non-negative values.

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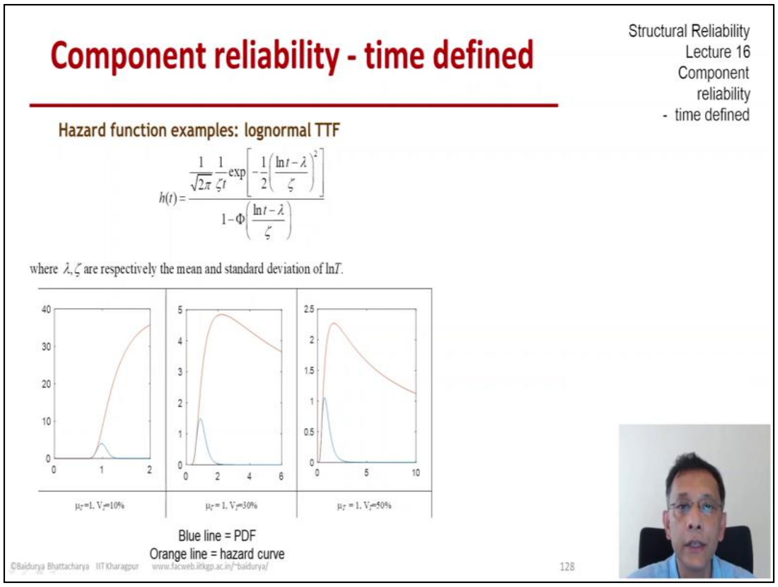

So, here we have the hazard function defined as a PDF over 1 - CDF as before and this particular function has some interesting shapes. So, let us let us review that. So, here first what you see is a log normal random variable whose mean is one and whose COV coefficient of variation is 10% it is rather small. So, the distribution is not very spread out and there we see the blue line is the PDF and the orange line is the hazard function.

So, clearly the hazard function is an increasing type and. So, it behaves similar to that of the normal TTF. So aging phenomena would be a good sort of basis for taking up the log normal t j but we have to be careful if the coefficient of variation increases. So, we have more scatter in the time to failure then an interesting thing starts happening here you see that the coefficient variation is 30% and the orange line the hazard function it increases quite sharply in the beginning but then it plateaus out and then it starts to fall.

So, you have to be careful before selecting a log normal TTF unless you know for a fact that the item in question in the beginning or for a large part of its life exhibits an increasing failure rate type behaviour and then suddenly it starts to have a decreasing failure rate. So, as long as that property is satisfied by the item is in question you can adopt a log normal TTF with a large 30% COV otherwise some other time to failure distribution should be preferred.

And this behaviour is even more pronounced when we have an even larger COV here you see 50% COV and you see the more pronounced behaviour of first increasing and then decreasing failure rate.