Structural Reliability
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Lecture –132
Component Reliability - Time Defined (Part - 11)

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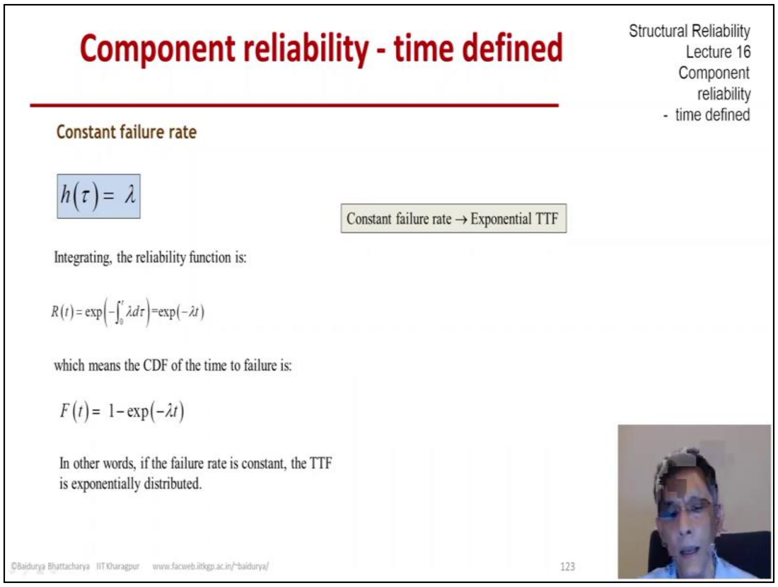

The hazard function can give us useful information about the local in time performance or state of health of the unit in question we can understand the behavior the likelihood of safe performance of a unit by looking at the shape of the hazard curve and in the next few slides we are going to keep coming back to that. So, we could have a hazard curve that was increasing with time we could have for some other item the hazard curve could be decreasing with time or it could stay constant with time not change or there could be combinations of this behaviour in different regions of time.

And one example that we are going to look at the end would be what is known as the bathtub curve which features all three that you see in the top are increasing decreasing constant. So, first let us take a look at the constant hazard curve or the constant failure rate function. So, it is it is a very important type of hazard curve and we are going to find out y in a minute. So, h of t h of tau is a constant lambda. So, lambda is independent of time.

So, we can integrate this and exponentiation and obtain the reliability function. So, R of t is exponential of minus integral lambda t tau from 0 to t and it's exponential of minus lambda t. So, this should be familiar if this is the reliability function then the CDF is 1 minus exponential minus lambda t and we immediately identify that this is the exponential random variable. So, the time to failure is an exponential random variable if the failure rate if the hazard functions is a constant.

So, this tells us that if an item does not become more likely or less likely to fail as it as time progresses which is a very special property it necessarily gives rise to an exponential time to failure and it. So, happens that many types of items like electronic items can exhibit fairly accurately a constant failure rate and for such item it is quite common to use the exponential time to the failure and that is what we see an abundance of sometimes maybe the exponential time to failure distribution is adopted rather hastily.

But it is it is a very common and it is a very useful distribution to have and when we do systems when we look at systems and other behavior the exponential time to failure for an element is very useful and it's a good simplifying assumption. So, to summarize if there is a constant failure rate for an item the time to failure is exponentially distributed. Now let us proceed with this idea. So, let us look at a series of this exponential time to failures and we have looked at this in the previous lecture where we had a Poisson process occurrence of shocks.

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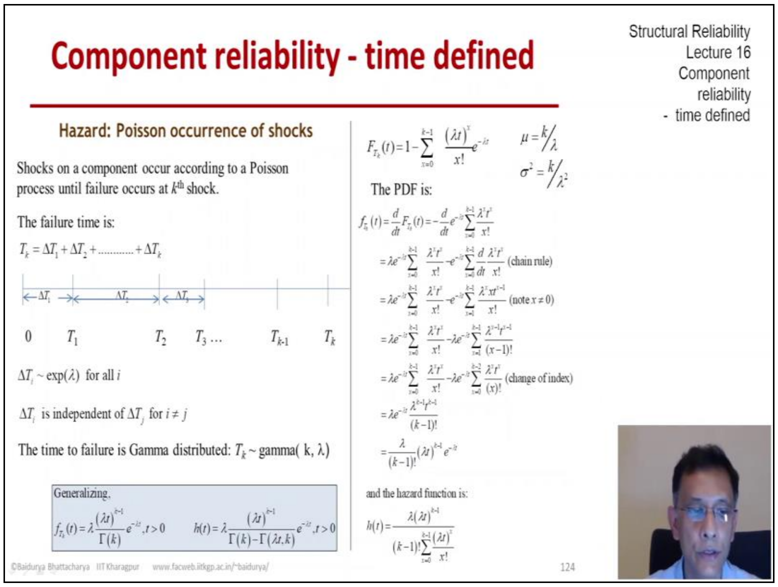

So, what we had was shocks occur according to a Poisson process until failure occurs at shock number k and that is the pictorial representation that delta T 1 delta T 2 and so on. So, the entire delta T's are independent and identical exponentials and once k of them occur we have failure. So, the time to failure T k is the sum of all these IID exponentials. So, as I said the delta T's are all exponential with the same rate lambda and they're mutually independent.

So, we derived last lecture that in this situation the time to failure T k is a gamma random variable with a density function CDF that we have looked before we have we have that list of density and distribution functions. So, if you need to look that up please do. So, so that now with this information let us see if we can find out what the hazard function corresponding to this failure model that failure occurs in shock number k when the shocks occur according to a parson process what that would look like.

So, what the hazard function looks like for a gamma distributed time to failure. So, let us proceed step by step this is the CDF of T k which we derived already in a previous lecture and I have listed the mean and variance for convenience. So, now let's derive the PDF of this random variable tk. So, the PDF is the derivative of the CDF now we have to proceed a little carefully because k terms are involved in the summation. So, we first use the chain rule of differentiation.

So we have two sets of terms there now we have to make sure that when we when we differentiate the sum the first term the exponent of T is zero. So, that cannot be differentiated with respect to time it is a constant. So, our index for x in the second summation starts from one instead of zero while for the first summation it starts from zero. So, once we do the algebra and undertake a change of index.

So, both of the sums now start from x equals 0 but the first one ends at k - 1 while the second one because the change of index now ends at k - 2. So, we essentially are left with the last term in the first series in the first sum and the density function is lambda times lambda t to the power of k - 1 exponential minus lambda T divided by k - 1 factorial. So, now with the PDF thus derived and the known CDF that we have.

So, we are in a position to define the hazard function of the gamma distributed time to failure. Obviously we are working with k being an integer here which is more appropriate for an Erlang distribution but if we generalize this then the CDF and the PDF they all involve the gamma function instead of the factorial. So, what you see on the screen on the left with the blue background is the density function and the hazard function when k is not necessarily an integer. So, now let us see what sort of shape this hazard function has.

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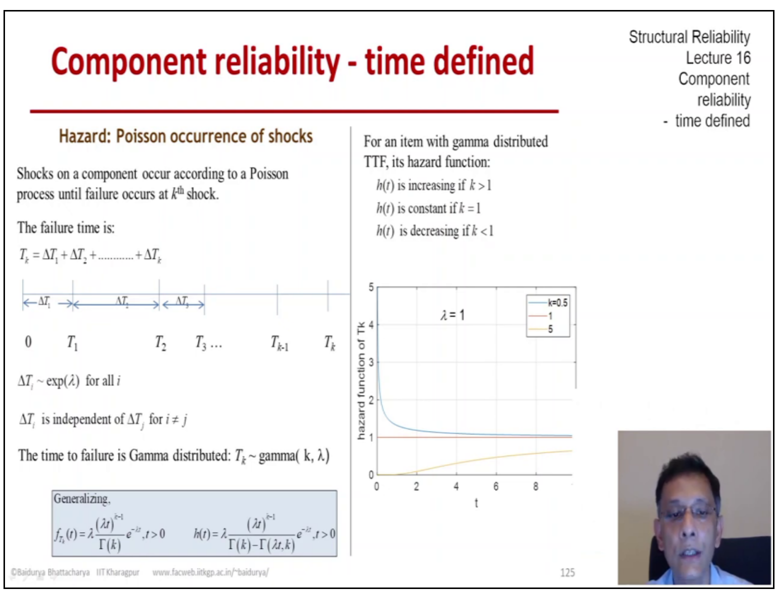

So, now depending on k it is interesting that the hazard function could be increasing it could be decreasing or it could remain constant obviously the constant would occur corresponding to the first failure. So, essentially what we are saying is the time to failure is exponential and we already have looked at in the previous slide that if the time to failure is exponential then the failure rate is constant.

So, there is no surprise there but if we just show with the plots. So, we can see the behaviour of the hazard function depending on whether the k whether k is greater than 1 or less than 1. When it is greater than 1 the example here you see is 5. So, it is an increasing function when k is less than 1 it is a decreasing function and as I said when k is equal to 1 we are left with an exponential time to failure and it is a constant function.