Structural Reliability
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Lecture –130
Component Reliability - Time Defined (Part - 09)

In the previous lecture we discussed the reliability function with the help of the random time to failure and what the reliability function does is it gives the likelihood that the item has survived up to time t. There is another way of looking at the items performance is to find out the likelihood of failure not up to time t but at or near about a particular instant of time t. So, the hazard function does that instead of giving an aggregate probability that the item will survive from zero up to the time in equation.

The hazard function gives the likelihood that the item will fail at time t but with the twist that the item has survived until that instant. So, let us now define the hazard function.

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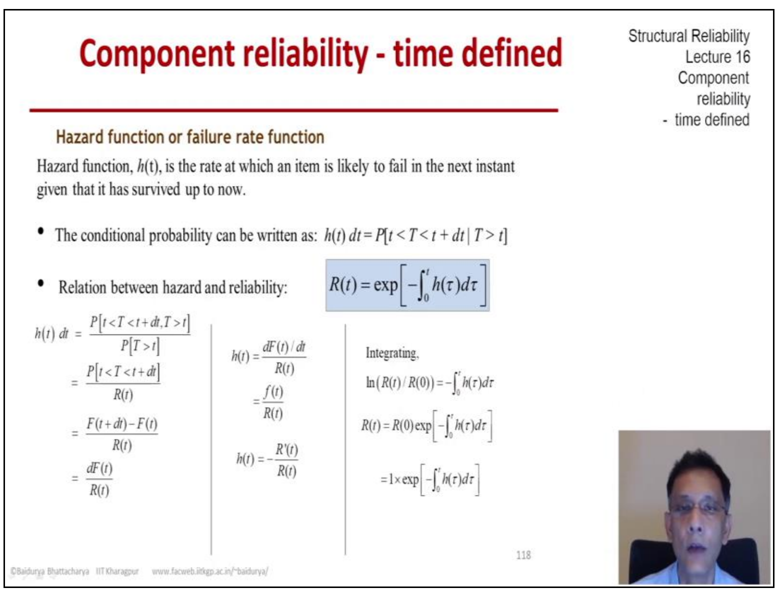

So, the hazard function is the probability that an item will fail in the next instant given that it has survived up to. Now as I said and expressing that as a conditional probability it looks like P that the random time to failure capital T is between small t and t + dt given and that is the conditioning event that capital T has survived up to little t. So, capital T is greater than t and this is the hazard function times the small increment of time.

So, ht dt is a conditional probability. Now let us see if we can find the relationship between the hazard function and the reliability function. So, let us start from the definition. So, ht dt is the conditional probability as you see on the screen. So, we can express the conditional probability P of a given b as P of a b over P of b and that is what we have done here. If you look at the numerator then the intersection of the two events that capital T is greater than small t and capital T is between small t and small t + dt is the second event that I just mentioned.

So, the numerator becomes P of capital T between small t and small t + dt and in the denominator the probability that the time to failure is greater than small t is nothing but by definition a reliability function. So, now let us focus on the numerator and that turns out to be the small increment in the CDF. So, the probability that a random variable is in a small region defined by t and t + dt would be the difference of the CDF at those two limits.

So, f of t + dt capital f minus capital F of t capital F is the CDF of the time to failure we have suppressed the subscript capital T because there is no scope of confusion here. So that is d of Ft divided by Rt is ht dt that is what we started with. So, we can express ht as a differential of the CDF over the reliability function and the differential of the CDF is the PDF. So, ht is the is the PDF divided by the reliability function.

So, the hazard function in one explanation is the density function divided by the reliability function gives us the same information that the item is likely to fail how likely it is to fail in the next instant given that it has survived up to now the density of that. So, let us see if we can use this relationship to actually eliminate the PDF and express h in terms of R or R in terms of h turns out we can f of t the PDF of t small f of t is nothing but the derivative of capital Fwhich would be the negative of the derivative of R.

So, ht is minus R prime t over Rt where the prime indicates the first derivative. So, now we are in the position to integrate this and we have done this in the past in high school and in first year of college. So, the integration is log of Rt over R0 R and the initial time that is the negative of the integral of h between 0 and t. And since reliability by definition at t equals 0 is 1. So, we have R of t is 1 times exponential of the negative of the integral of ht.

So, it is the area under the hazard curve takes the negative of that exponentiation that that would be the reliability function. So, that's how the hazard and reliability functions are related.

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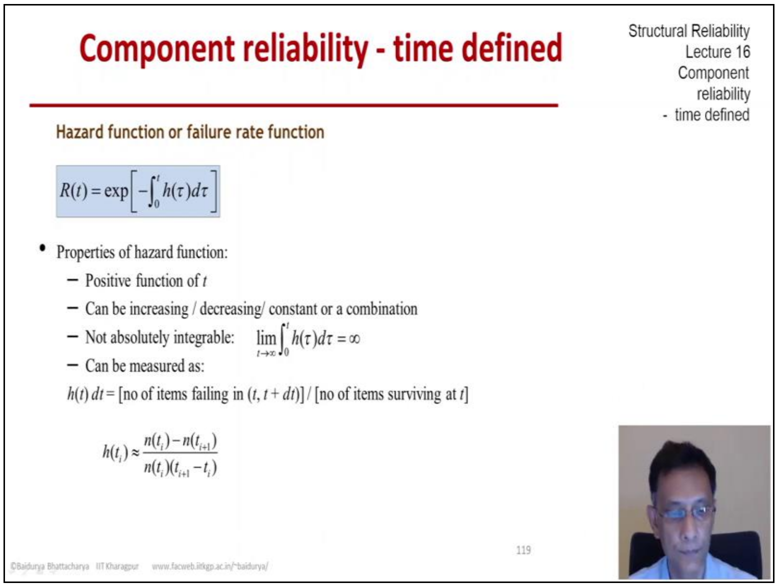

If we; now let us look at the properties of the hazard function. So, obviously it is the ratio of two positive functions. So, it is itself positive for all t and t is defined over 0 and higher values. It could be a decreasing function it could be increasing function it could be a constant or a combination of any of those it does not have to be monotonic in any sense but a very important property is that it’s integral between 0 and infinity must blow up.

So, the integral cannot be a finite number. So, that is also an essential property for a function to qualify as a hazard function. And that should be obvious because at infinity the reliability function has to be 0. So, that is why we must have this property of the hazard function. We can measure it we can measure it as by definition there are other measures also which we will come to later in this lecture but we can start with the definition that ht dt is roughly the number of items failing in a window of time t and t + dt divided by the number of items that survived at this point have survived up to now.

So you can approximate that in a testing program as the ratio of the number of items failing between t i and t i + 1 divided by the length of that window t i + 1 minus t i and the number of items that survive at t i. So, that would be an approximate value of hazard function from a test program.