Structural Reliability
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Lecture –13
Review of Probability Theory (Part -05)

The third axiom of probability tells us that the probability of the union of 2 disjoint sets A and B is the sum of the individual probabilities. Now what if A and B are not disjoint.

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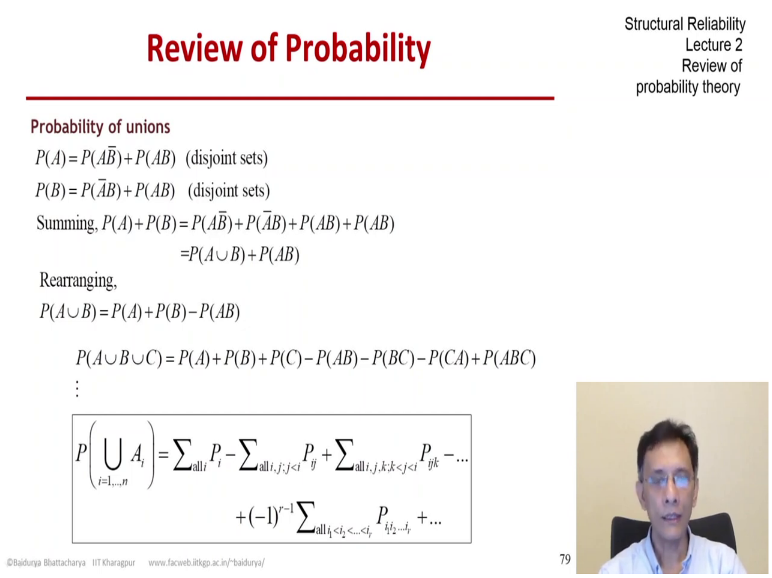

So, let us express the event A as the union of 2 disjoint sets. So, that would be A B complement and AB. So, I can write the probability of A as the sum of these 2 probabilities that you see on the screen and likewise I can express the probability of B as A sum of 2 probabilities as well. Now I can sum these 2 lines and after just rearranging I come up with the probability of A union B when they are not necessarily disjoint.

Now this can be easily extended to the union of three events A B and C and you can see a pattern emerging that I first add all the first order probabilities then I subtract the second order joint probabilities and then I add the third order probability and so on. So, that gives me in the case of the union of n events the inclusion exclusion formula. So, I go through all orders of joint probabilities until I complete them. Now is there a way to express the intersection probabilities in some convenient manner?

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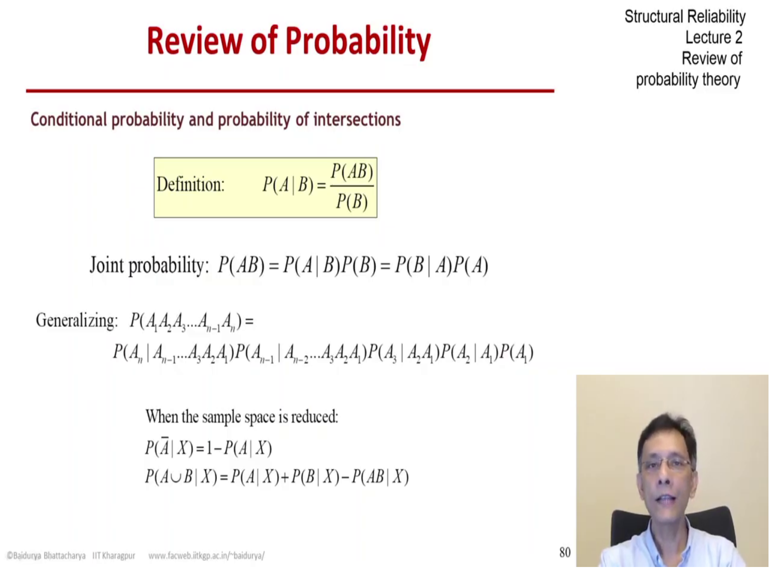

For that let us define the conditional probability in some sense the conditional probability is restricting the sample space. So, what is the probability of A given that B has occurred. So, it is defined as the ratio of P of A intersection B and P of B. So, that lets me to express the permutable joint event AB as the product of one conditional and one marginal. This can be generalized to in the case of n events like in a chain that you see on the screen as the product of conditional probabilities all the way up to P of A1.

Now one point to note is that when we reduce the sample space this way all the known rules still hold. So, if P of A complement is 1-P of A then I restrict the sample space to X. So, E of A component given X would still be 1-P of A given X and likewise P of A union B given x would still be P of A given X + B of B common X - P of AB X.

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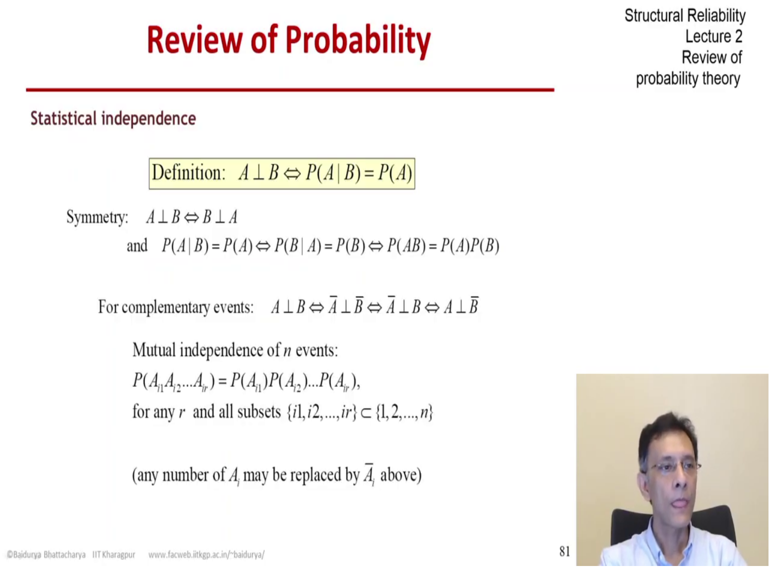

Let us move on. Now to the concept of statistical independence, now independence is a rather delicate concepts 2 events are said to be independent if they are not related in any way and neither causative nor associated. Now often this assertion of independence between 2 events is made from experience and heuristic reasoning and not so, much from reverse proof. So, in many cases it is more a statement of faith.

Now 2 events are said to be statistically or stochastically independent if and only if the probability of occurrence or non-occurrence of one is not affected by the knowledge of occurrence or non-occurrence of the other. So, this is the symbol we use. So, if A and B are statistically independent then the necessary and sufficient condition is P of A given B is equal to P of A and I could write equivalent statements which you see.

So, in the end P of A intersection B is the product of PA and PB. When I have complementary events for example these would all hold. So, all these equations would hold equally well when A is replaced by A complement or B is replaced by B complement when I extend this to the mutual independence of N events. So, the probability of the joint probability of any subset of them must be the product of the individual probabilities. And as in the case of A and B this would hold if I replace any of these A's with its continuity.

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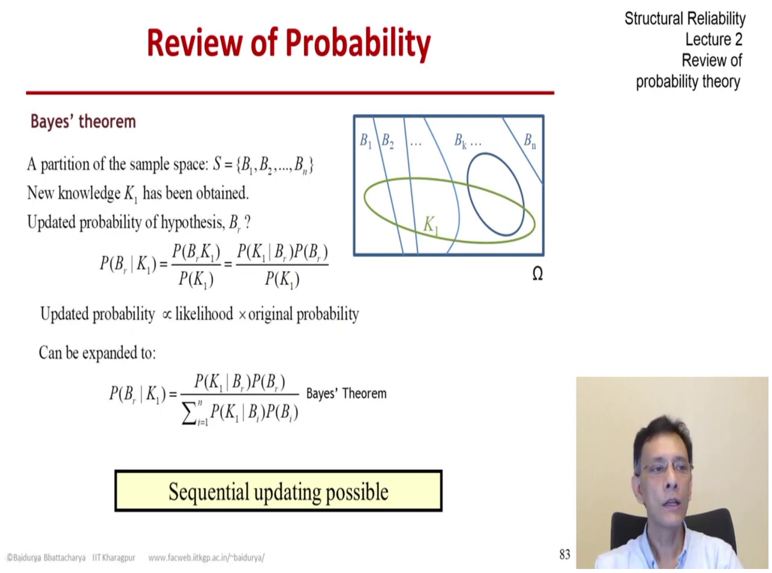

Now let us move on to the theorem of total probability we have partitioned the sample space into B 1, B 2 up to B n. So, the B's are collectively exhaustive and mutually exclusive. So, I can define a new event A on the sample space and for any such set A I can describe it as the union of its intersection with all the individual B's and because these intersections are disjoint I can conveniently express the probability of A as the sum of all the intersection probabilities.

Now this I can expand I can expand each of the joint probabilities AB i using the conditional. So, P of AB i is P of A given B i times P of B i. So, this is a more familiar expression of the theorem of total probability. This takes me to the Bayes theorem. So, suppose I have the same partition of the sample space B 1 through B n and suppose some new knowledge which I call K 1 has been obtained and say these B's are some original hypothesis.

So, with this new knowledge can I update the probability of one of these say B r it turns out that I can do it by simply restating the definition of the conditional probability. So, I can express P of B r given K 1 as the original P of B r times the probability that K 1 would occur if B r is true and normalized by the probability of K 1. So, this is basically stating that the updated probability of B r with the new knowledge K 1 is proportional to the likelihood of that knowledge and the original probability of B r.

A more familiar expression of Bayes theorem is what you see now where the denominator is expanded with the help of the theorem of total probability. Now if new knowledge keeps coming in K 1 K 2 and so, on sequential updating of P B r is straightforward.