

**Structural Reliability**  
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**Lecture –129**  
**Component Reliability - Time Defined (Part - 08)**

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## Component reliability - time defined

Structural Reliability  
Lecture 15  
Component reliability  
- time defined

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**Example:**

The fatigue life,  $N$ , of a structural weld under cyclic loading is given by  $N = KS^{-m}$  where  $S$  is the stress amplitude, and  $K, m$  are material and geometry dependent parameters.

$S$  and  $K$  are lognormal random variables (mutually independent) while  $m$  is non-random.

The mean and c.o.v. of  $S$  are 100MPa and 30% respectively. The mean and c.o.v. of  $K$  are  $10^{12}$  cycles and 20% respectively (when  $S$  is in MPa). The value of  $m$  is 3.

a) What is the distribution of the random fatigue life?

b) What is the probability that the component will survive 1 million cycles?

$$P[N \geq 10^6] = ?$$

$$= 1 - P[N < 10^6]$$

$$= 1 - \Phi\left(\frac{\ln 10^6 - 13.9}{.904}\right) = 0.55$$

$S \sim LN(100, 30\%)$

$\ln S \sim N(\mu_{\ln S}, \sigma_{\ln S}^2)$

$\sigma_{\ln S} = \sqrt{\ln(1+V_S^2)} = 0.294$

$\mu_{\ln S} = \ln \mu_S - \frac{1}{2}\sigma_{\ln S}^2 = 4.56$

$K \sim LN(10^{12}, 20\%)$

$\ln K \sim N(\mu_{\ln K}, \sigma_{\ln K}^2)$

$\sigma_{\ln K} = \sqrt{\ln(1+V_K^2)} = 0.198$

$\mu_{\ln K} = \ln \mu_K - \frac{1}{2}\sigma_{\ln K}^2 = 27.6$

$N = KS^{-m}$

$\ln N = \ln K - m \ln S$

$\ln N \sim N(\mu_{\ln N}, \sigma_{\ln N}^2)$


$\mu_{\ln N} = \mu_{\ln K} - m\mu_{\ln S} = 27.6 - 3 \times 4.56 = 13.9$

$\sigma_{\ln N}^2 = \sigma_{\ln K}^2 + m^2\sigma_{\ln S}^2 = .198^2 + 9 \times .294^2 = 0.817 \Rightarrow \sigma_{\ln N} = 0.904$

$N \sim LN(\mu_N, V_N)$

$V_N = \sqrt{e^{.817} - 1} = 112\%$

$\mu_N = \exp\left(13.9 + \frac{1}{2} \cdot .817\right) = 1.69 \times 10^6$



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In this example we generalize the idea of time to failure somewhat and instead of a rigid definition of time we bring in number of cycles. So, now we have a random number of cycles to failure we did look at an example of some tests a few lectures ago so it is looking at that in a more formal setting. So, let us take a minute to read the problem and let us solve it after that. So, what we see here is the stress amplitude that is a random variable the coefficient  $K$  is a random variable and the exponent  $m$  is a constant a non-random number.

In general  $m$  is also a random variable in such situations and it is correlated with  $K$  but if we bring that in it is going to just obscure a few things and make the computations unnecessarily demanding. So, we are just going to treat  $m$  as a non-random constant for now. So, this kind of gives an idea is the random time to failure. We are now talking about random number of cycles to failure random time to failure is parameterized by the stressor.

So, the stress amplitude here is the stressor and that is now considered a random quantity uh. So, this kind of gives an idea about what might be done in an accelerated test this formulation really does not make it more physics based we are still treating the fatigue cycling and fatigue failure as a black box we just have parameterized the problem with the random stress aperture. So, let us now look at the details S and K are log normal random variables.

So, that kind of points that we need to take log and see what we get. So let us get on with the solution S is not normal. So, log S is normal with mean  $\mu_{\log S}$  and variance  $\sigma_{\log S}^2$  and we know these formulas very well we can derive  $\sigma_{\log S}$  and  $\mu_{\log S}$  in terms of the numerical values and obtain the values as you see on the screen we can do likewise for the coefficient K and K is log normal therefore log K is normal and we have numerical values for  $\sigma_{\log K}$  and  $\mu_{\log K}$  with these we are able to find the properties of the random life the number of cycles to failure n.

So, taking log on both sides. So, log of n is a linear combination of log K and log s m being a constant. So, therefore we have a linear combination of normal which means log n is normal which means n is log normal. So, if it is log normal. So, let us find the corresponding normal quantities. So,  $\mu_{\log n}$  and  $\sigma_{\log n}$  and just by using the definition which we have looked at before the mean of log n and the variance of log n can be derived and that's what you see on the screen.

So, the mean of log n is 13.9 and the variance of log n is 0.904 and that actually would give us the values the mean of n and the coefficient of variation of n and which happen to be about 112 percent. So, very large coefficient variation and a mean life of about 1.7 million cycles and that n is a log normal; random variable. So, we are now in a position to solve part b the probability that the component will survive 1 million cycles.

So, what we are looking for is that P of n greater than or equal to 1 million and we can solve it take the complement of that make use the fact that n is log normal. So, log n is normal bringing in the normal distribution function and the final answer is 0.55. So, this would be a way of finding the fatigue reliability.