Structural Reliability
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Lecture –128
Component Reliability - Time Defined (Part - 07)

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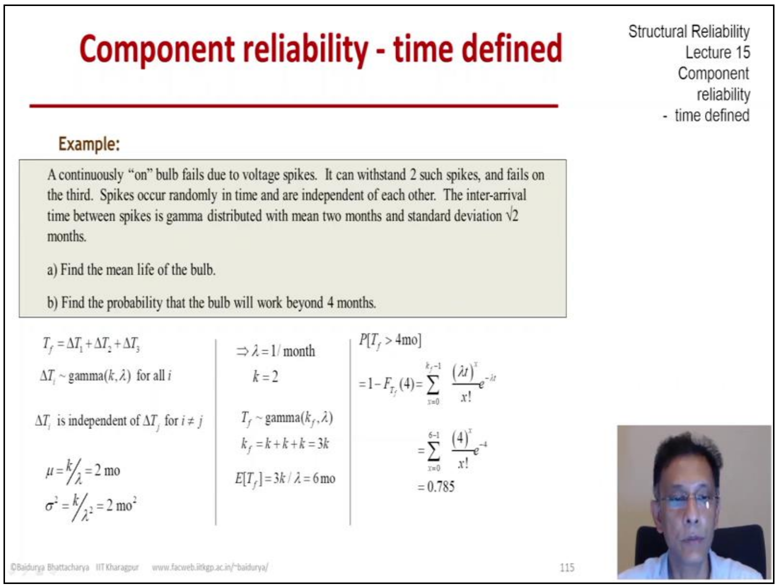

This example involves sums of independent gamma random variables let us take some time to read the problem and solve it after that. So, let us first start defining the time to failure for the bulb. So, the time to failure of the bulb is the sum of the inter-arrival times of the three spikes. So, delta T 1 + delta ,T 2 + delta, T 3 because the bulb fell fails on the third spike. So, it is the sum of three delta T's.

Now each of these delta T’s is gamma distributed and they are mutually independent we are going to use that very soon uh. So, delta T i is gamma with parameter k and lambda and that is true for all i. So, i going from 1 to 3. They are independent they are mutually independent and for each delta T the mean is 2 months and the variance is 2 1 squared just to be dimensionally consistent. So, this lets us find the value of k and lambda and we can derive that lambda is 1 per month and k is equal to 2.

So, that is how I am able to fully define each of these inter arrival times delta T with k= 2 and lambda equals one per month and they are mutually independent gamma random variables. Now the sum of the gamma random variables as we saw in a couple of slides back that the sum of independent gammas is also gamma provided the rate remains the same which means lambda is unchanged.

So, now I have T f being the sum of three independent gamma random variables with the same lambda i can conclude that T f is again gamma distributed with a lambda as before and k f is the sum of the constituent gammas. So, it is because all the gammas if the sum of the constituted case. So, since all the k's are the same. So, k f is 3 times k. So, now I have a new gamma random variable T f with parameters 3k and lambda.

So, we let us find out the mean life of the bulb. So, the mean life of the bulb the mean of the gamma random variable is k over lambda. So, in this case it is 3k over lambda and that is 6 months. So, the average life of this bulb is 6 months and now we need to find out the probability that this pulp whose mean is six months is going to work for at least four months. So P of T f is greater than 4 months.

Let us go back to the basics the way we defined the CDF of the gamma random variable which we did couple slides back. So, 1 - CDF of the gamma random variable is actually what we want evaluated at 4 months that is given by the sum of a few Poisson PMFs. So, X going from k - 1. So, here lambda is one per month as before T is 4 months k f is thrice k which is 6 and we need to add these 6% PMF terms and if we do that right then the answer comes to 78.5%. So, that is the probability that this particular bulb will work for at least 4 months.