

Structural Reliability
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Lecture –127
Component Reliability - Time Defined (Part - 06)

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Structural Reliability
Lecture 15
Component
reliability
- time defined

Component reliability - time defined


Example:

The one-month reliability of an indicator lamp is 0.95. The lamp has a constant failure rate. What is the probability that more than two spare bulbs will be needed during the first year of operation? (Ignore replacement time).

$CFR \Rightarrow$ exponential TTF $\Rightarrow R(t) = \exp(-\lambda t)$
 $R(1\text{mo}) = 0.95 \Rightarrow \lambda = 0.0513 / \text{mo}$
[more than 2 spare bulbs needed in 12 mo] = [more than 2 failures in 12 mo] = $\{N(12\text{mo}) > 2\}$

$P\{N(12) > 2\} = 1 - P\{N(12) = 0\} - P\{N(12) = 1\} - P\{N(12) = 2\}$
 N is a Poisson random variable

$P\{N(12) > 2\} = 1 - \exp(-12\lambda) \left[1 + \lambda t + \frac{(\lambda t)^2}{2} \right] = 0.025$



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Let us solve a problem involving occurrence of events on the time axis according to a person process let us take a minute to read the problem and then we will solve it. So, the key point to note here is that word constant failure rate. So, that we are going to see very soon actually indicates that the process occurs according to person process with a constant rate that constant rate is lambda. So, and the time to failure is distributed according to an exponential random variable.

So, the probability that more than two spare bulbs will be needed. So, we are looking at sums of independent exponentials the gamma Erlang distribution which we are going to solve using the Poisson PMF all of which we derived in the previous slide. So, the constant failure rate implies as I said the exponential time to failure which implies that the reliability function is 1 minus the CDF of the exponential random variable which is simply exponential of minus lambda t.

So, the reliability since it is given its 0.95 at one month we can deduce the lambda from there which turns out to be about 0.05/month. Now the next thing that we need is the event that more than two spare bulbs will be needed in a period of 12 months. So, that means there will be more than two failures in those 12 months which basically means is that number of occurrences in those 12 months the number of person occurrences in those 12 months will be more than 2.

So, we can find the probability of that event. So, P of n in 12 will be greater than 2 is the same as 1 minus the probability that there will be 0 or 1 or 2 events in that interval of 12 months and now we recall the property that n is a Poisson random variable because the underlying process is a Poisson process with rate lambda and we are now in a position to put in the PMF formula for the personal random variable and we find the answer as about 2.5%.