Structural Reliability

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Lecture –126
Component Reliability - Time Defined (Part - 05)

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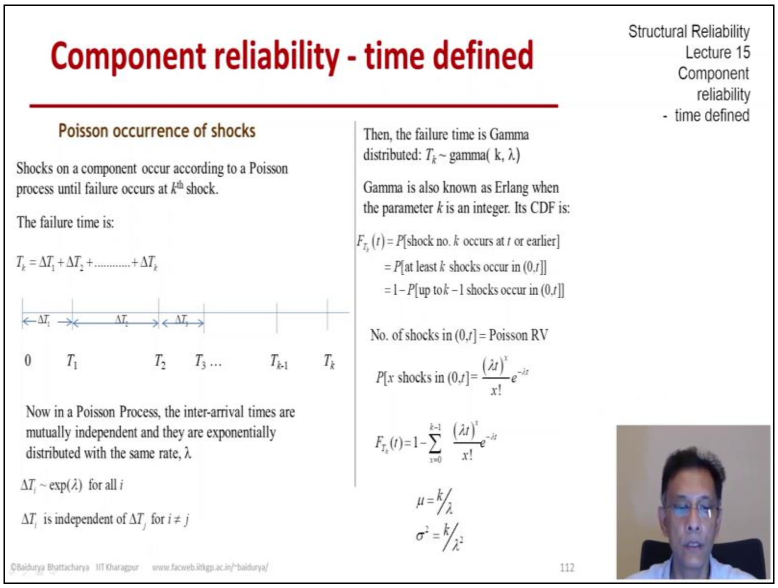

Let us look at random events occurring on the timeline leading to failure of an item in some detail. So, let us say that we have a component which has some reserve strength and it can withstand some shocks not fail at the first shock itself and these shocks occur according to a Poisson process. So, this component fails at shock number k we did look at the person process briefly when we discussed joint distributions.

So, this is a revisit to Poisson processes. The failure time would be the sum of all these inter arrival times. So, if failure occurs at shock number k the time is built up with increments delta T 1 delta T 2 all the way up to delta T k each such increment is the inter arrival time between successive shocks. So, pictorially speaking this is what we have we start from 0 and then after delta T 1 the first shock occurs. So, that is at random time T 1 then after random time delta T 2 after random interval delta T 2 the second shock occurs at random time T 2 and so, on until we reach the k^{th} shock occurring at a T k.

So, we know that in a Poisson process the inter-arrival times are exponentially distributed with the rate lambda and they are mutually independent. So, that is a very useful and very powerful property and to reiterate that. So, all the delta T's are exponential with the same constant lambda which is the rate of the Poisson process and they are mutually independent in disjoint intervals. So, the failure time, so, the sum of these independent exponentials is a gamma random variable and T k is gamma distributed the gamma distribution is defined by two parameters k and lambda.

So, if k is an integer it is sometimes known as the Erlang distribution. Now let us derive the CDF of the gamma random variable. So, by definition the CDF of T k evaluated at small t is that the shock number k occurs at time T or earlier. So, the probability of that that event which we can restate as the probability that at least k shocks occur in the interval 0 to T and if we take the complement it is 1 minus the probability that up to k minus 1 shocks occur in interval zero to t.

So, zero shock or one shock or two all the way up to k minus one shock can occur in the interval 0 to t. So, one minus the probability of that, would give us the CDF of the failure time t k. Now this is also a very useful result comes out of the Poisson process is the number of events in a Poisson process over time 0 to t is actually a Poisson random variable. And we have seen the Poisson random variable extensively when we discussed the common distribution functions.

So we should be able to use the Poisson PMF and the person CDF. So, that x shocks occurs in interval 0 to t we know the PMF it is lambda t to the power x over x factorial times exponential of minus lambda t the lambda is the same lambda that defines the Poisson process. So we need to add x from 0 to k - 1 and subtract that sum from 1 that would give me the gamma or more specifically the Erlang CDF and its mean and variance could be defined in terms of k and lambda and that is what we have here the mean is k over lambda and the variance is k over lambda squared.

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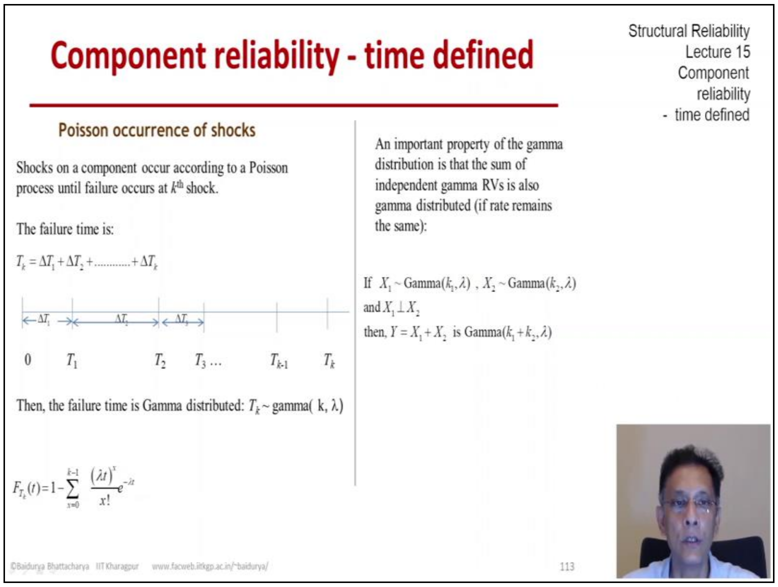

Now continuing the gamma distribution is closed under addition the gamma family provided the rate does not change. So, if we continue adding gammas from the same underlying process defined by lambda gamma k 1 lambda and gamma k 2 lambda if they are independent then their sum would be a new gamma random variable with parameter k 1 + k 2 and the same lambda as before. So, now let us solve one or two examples with this sort of arrival on the time axis.