

**Structural Reliability**  
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**Lecture –124**  
**Component Reliability - Time Defined (Part - 03)**

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**Component reliability - time defined**

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**Example:**

A manufacturer of display units determines that the units are used on average 1.8 hrs a day. A one-year warranty is offered on the display unit. The unit has an exponentially distributed life with an MTTF of 2000 hrs. Find what fraction of the tubes will fail during the warranty period.


$T \sim$  exponential (2000 hrs)

$$F_T(t) = 1 - \exp(-t / \mu_T)$$
$$t = 365 \times 1.8 \text{ hr} = 657 \text{ hr}$$

Fraction failed =  $F_T(657 \text{ hr})$

$$= 1 - \exp(-657 \text{ hr} / 2000 \text{ hr}) = 0.28$$

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Let us solve some problems involving time to failure based approach to the reliability function let us take a minute to read the problem and then we will solve it step by step. So, let us define the random variable capital T as the time to failure exponentially distributed with a mean of 2000 hours the exponential random variable has a CDF which looks like this where the mean mu is the required parameter. So, what is asked for is to compute the CDF at a certain time.

So, let us find small t first and 1.8 hours a day over a year which is the design period which is the warranty period and then so, that comes to about 657 hours. So, the fraction of tubes that will fail during the warranty period the fraction of display units is the CDF of capital T evaluated at that particular time instant which is 657 hours. So, if we just plug in the values it comes to 0.28. So, 28% of all such display units would be expected to fail before the end of the warranty period.

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## Component reliability - time defined

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### Example:

The CDF of the TTF of a component is:  $F(t) = 1 - e^{-0.04t - 0.008t^2}$

Specify the design life so that reliability never falls below 90%

$$R(t) = 1 - F(t)$$

Find minimum  $t^*$  such that  $R(t^*) \geq 0.9$

$$\Rightarrow 0.04t^* + 0.008t^{*2} = 0.10536$$

$$\text{or, } t^{*2} + 5t^* = 13.17$$

$$\text{or, } (t^* + 2.5)^2 = 19.42$$

$$t^* = -2.5 \pm 4.41$$
$$= 1.91$$



The next problem that we will look at involves starts with the CDF of the time to failure of a certain component. So, it is given as you see on the screen and we need to find out the design life such that it never falls below 90%. So, if you would like to solve it please pause the video otherwise I will present the solution step by step. By definition the reliability function is 1 minus the CDF of the time to failure and we need to find the minimum time  $t^*$  such that  $R$  at  $t^*$  is at least 0.9.

So, it now becomes a simple solution of  $t^*$  from one variable equation. So, we end up with a quadratic equation in  $t^*$  and because  $t$  is only non-negative the time to failure is defined only for non-negative values so the answer is 1.91 units of time. So, that is the time up to which the reliability is always greater than or equal to 0.9. So, beyond 1.9 units of time the reliability is going to fall below the specified 90%.

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## Component reliability - time defined

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### Example:

The pdf of the time-to-failure of an appliance is

$$f_T(t) = \frac{32}{(t+4)^3}, \quad t > 0$$

- Find the reliability function of the appliance.
- Specify the design life so that reliability never falls below 80%
- Find the MTTF.
- What is the MRL at the design life?

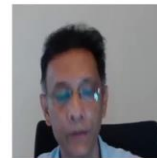
$$F_T(t) = \int_0^t \frac{32}{(x+4)^3} dx = -16 \left[ \frac{1}{x^2} \right]_0^t = 1 - \frac{16}{(t+4)^2}, \quad t > 0$$

$$\Rightarrow R(t) = \frac{16}{(t+4)^2}, \quad t > 0$$

$$R(t^*) = 0.8, \quad t^* > 0 \Rightarrow t^* = -4 + \sqrt{20} = 0.47$$

$$\mu_T = \int_0^{\infty} R(t) dt = \frac{16}{-1} (t+4)^{-1} \Big|_0^{\infty} = 4$$

$$\mu_{T|t^*} = \frac{1}{0.8} \int_{t^*}^{\infty} R(t) dt = \frac{16}{0.8} (t+4)^{-1} \Big|_{t^*}^{\infty} = 4.47$$



The next problem we look at again starts with the definition of the density function of the random time to failure. So, our starting point is the PDF and we need to find out four answers. So, let us take a minute to read the problem and then we will take up the solution. So, first we find the reliability function of the appliance let us start with finding the CDF which is the integration of the density function and we integrate the density function which is defined over the entire line from 0 to infinity.

So,  $x$  is the dummy variable and we integrate that and we obtain the answer as  $1 - 16$  over  $t + 4$  whole squared so that is for all non-negative values of  $t$ . So, that is the CDF of the time to failure and we know that the reliability function is 1 minus this CDF. So, the reliability function is  $16$  over  $t + 4$  whole squared as you see on the screen. So, the next two parts part c and d asks us to find the mean time to failure and the mean residual life at the design life which is we have to find next.

So, the design life as we did in the previous problem is that point in time at which the reliability equals 0.8. So, we need to solve for  $t^*$  and the answer is about 0.47. So, that is the only feasible solution from that quadratic equation uh. So, at zero point we will find out the mean residual life but let us first find the mean time to failure. So, the unconditional mean. So, that would be by definition we will use the area under the reliability curve that is convenient.

And if we complete the integration we get an answer of 4 units of time now let us plug in these quantities in the definition of the mean residual life. So, the mean residual life integrates the reliability function to the right of  $t^*$  and normalizes by the reliability at  $t^*$ . So, this is by definition 0.8. So, if we complete the integration we get an answer which is 4.47. So, it is interesting to note that the mean residual life at the design life is greater than the original mean time to failure and that is that might appear surprising.

But it is a particular kind of component that we will find out in the next lecture has what is known as a decreasing failure rate. So, components that exhibit such kind of property would have the mean residual life like you see here which is a little larger than the unconditional mean life.