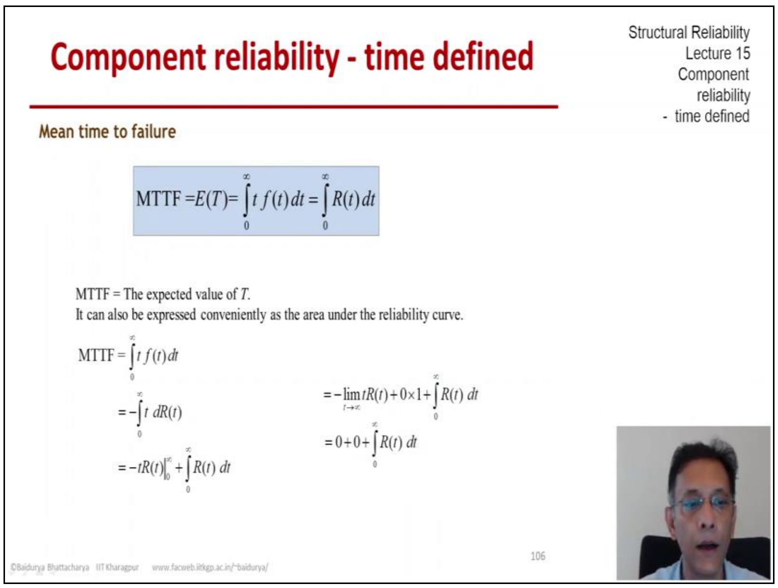
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –123
Component Reliability - Time Defined (Part - 02)

The mean time to failure is a very useful and popular measure of reliability especially for electronic components machine parts and not so, much for structural members for building systems and so on. Mainly because in structural engineering the failure probabilities are so, rare that it is not. So, much practical to talk about the mean time to failure of a building or a bridge but let us define it.

(Refer Slide Time: 01:07)



The MTTF the mean time to failure is the expectation of random time to failure t and because we define t only for non-negative values 0 up to positive infinity. So, that is what you see on the limits of the integration but that also gives rise to which we will prove next to the last equation on the right which says that the mean time to failure of a component is the area under its reliability curve. And which is actually a very useful property to have in some situations.

Such as when in a test program not all the members that we start with not all the specimens that we start with fail before the test program is over. So, there are many specimens which last longer than the duration of the test. So, if we estimate the mean time to failure as just the average of the observed times to failure then obviously that would be an underestimate of the actual mean. So, in such cases it is much better to estimate the reliability function and extrapolate it beyond the time of the test program and then estimate the area under it.

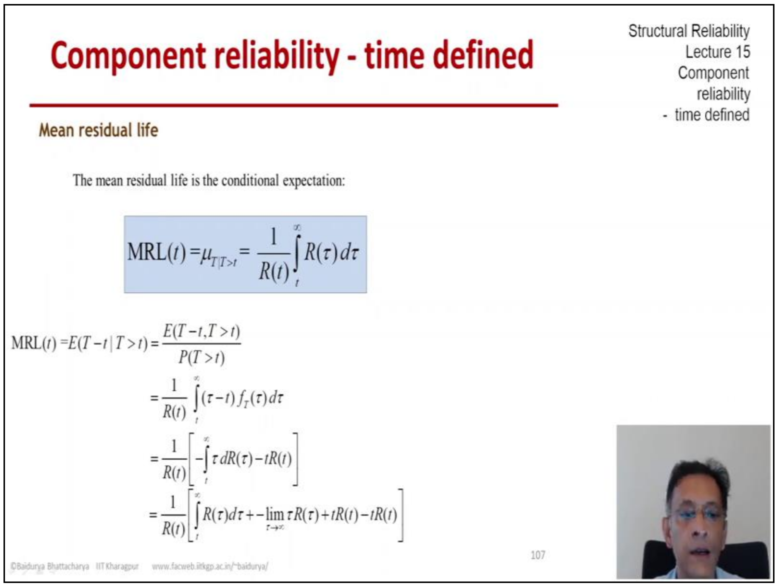
So, that would give a closer value of the MTTF. Now let us see if we can we can prove how the last equation comes that the MTTF is the area under R of t. So, by definition MTTF is as you see there the integration of t f t over the entire range now by definition f of t the density function of t and the reliability function of t are related that the density function is the negative of the derivative of the reliability function.

Why, because the reliability function is 1 minus the CDF and the density function is the derivative of the CDF. So, the density function is the negative of the derivative of the reliability function. So, that is how you see MTTF is also negative of t dRt when the limits on t are 0 to infinity. Now we can expand this by integration of by integration by parts and so, the first one is t R evaluated at zero and infinity and the second one is the area under the reliability curve from zero to infinity.

So, the first term we've evaluated at infinity. So, t Rt when t approaches infinity and the other one is t Rt evaluated at 0 which obviously is 0 times 1 because at t equals 0, R starts at the value 1. So, but what happens to the first term the limit of t Rt as t goes to infinity it. So, happens and which we will clearly see when we define the hazard function is R of t goes down faster than t. So, t of t times Rt as t goes to infinity approaches 0.

So, that first term ends up being zero. So, we are left with the area under the reliability curve that we showed at the beginning.

(Refer Slide Time: 05:29)



A related measure coming out of the random time to failure is the mean residual life. So, we now expand the definition of the mean time to failure as a conditional. So, it is the mean time to failure given that the structure the system the item has survived up to the present instance. So, capital T greater than little t based on that event conditioned on that event I want to find the mean. So, it is a conditional mean that we are looking at and we have defined and we have worked with conditional means before when we are discussing joint distributions and etcetera.

So, this by definition we can show in a minute we will prove it but this ends up being the area under the reliability curve from t onward and normalized by the reliability at t. So, let us see if we can prove it. So, this is our starting point the definition is that the mean residual life at small t is the expected value of the remaining life. So, capital T minus small t on the condition that the capital t is greater than small t. So, we use the definition of a conditional probability that you know E of a given b.

So, we express that as the ratio in the denominator you have the probability that capital T is greater than small t which by definition is the reliability at small t. So, we have figured out that part and now let us focus on the numerator the numerator would be the integration from t to infinity of tau minus t times f t dT. So let us expand that we can go through the algebra and it comes out looking something similar which we have already seen in the case of the MTTF is we have within the brackets the area under the reliability curve going from t to infinity.

And then the limit of tau r tau as tau goes to infinity and then t Rt added and subtracted. So, they cancel out each other and what we are left with is by the same logic limit of tau r tau as tau goes to infinity it vanishes because our tau falls off faster than one over tau and then we are left with the expression that we started with and the mean residual life is the area under the reliability curve starting at t.

So, to the right of t and normalized by the reliability at that instant, just as a check when t is equal to 0 we are left with just the mean time to failure as it should be.