

**Structural Reliability**  
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**Lecture –120**  
**Time Dependent Component Reliability (Part - 01)**

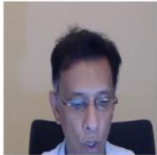
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Structural Reliability  
Lecture 15  
Time dependent  
reliability

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- Part A
  - Motivation
  - Basics of probability
  - Basics of random variables
  - Common probability distributions
  - Joint distributions
  - Monte Carlo simulations - discrete continuous and dependent variate generation
- Part B
  - History and scope of reliability studies
  - Definition and terminologies
  - Reliability problem formulation
  - System representation & redundancy(so far)

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With this lecture we are going to start our discussion on time to failure based description of reliability both for components and systems but let us set the stage first. So, this is what we have done in the course. So, far in part a we discussed the motivations for this course I went through the basics of probability theory the basics of random variables described a few distributions both discrete and continuous that occur commonly in the subject.

Joint distributions, convergence and introduced Monte Carlo simulations particularly generation of discrete continuous and dependent variants. In part B which is going on now we introduced the history how the subject developed the scope of reliability studies defined certain common terms. Discussed how reliability problems can be formulated and described how a system can be represented in terms of its elements and also had some words on the concept of redundancy.

We are now in a position to discuss in detail what makes structural reliability problems a little bit

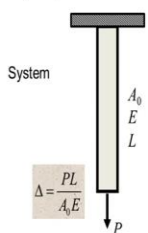
different from what is usually tackled in general reliability engineering.

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## Recap: Structural reliability problem formulation

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**Key steps**



$\Delta = \frac{PL}{A_0 E}$

Displacement limit state:  
**{Failure} =  $PL / A_0 E > \Delta_{max}$**

- Define
  - System and its behaviour ("elastic cable carrying a specified load")
  - Its performance objective(s) ("must carry applied load with small elongation")
  - Limit(s) of satisfactory performance ("elongation at most  $L/1000$ ")
- Identify:
  - Relevant system properties ( $A_0, E, L$ )
  - Relevant input(s) ( $P$ )
  - Response(s) of interest ( $\Delta$ )
  - All relevant probabilistic information (random variables, random processes etc.)
- Create appropriate system (I/O) model:  $\Delta = f(P; A_0, E, L)$
- Express failure condition ("limit state")
  - in terms of system capacity(ies) and system response(s) ...
  - as a precise mathematical statement (usually involving one or more inequalities) ...
  - corresponding to each performance objective



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So, let us go back to the key steps that we discussed at the beginning of this part that we have a system which is defined in terms of its behaviour its performance objective the limits of satisfactory performance we are able to identify the system properties the inputs and the responses of interest and the corresponding probabilistic information. And then we are able to create the appropriate model for the system and then which is what we have been spending a lot of time on in this part of the course is express the failure conditions.

The limit states in terms of system capacities and system responses as a precise mathematical statement for each of these performance objectives and then define the whole system. So, this was the reason that we discussed the problem formulation and how to represent system in terms of its elements.

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## Recap: Structural reliability problem formulation

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**Cantilevered beam with three failure modes**

The structural system, its loads and failure modes:  
 The point load,  $P$  (time-varying), and the uniform load,  $W$  (time-invariant), are acting together.  
 Beam can fail in three modes:  
 Flexure (at B)  
 Shear (at B)  
 Excessive deflection (at C)

$$Rel(t, \Omega) = P[C(\tau, \underline{x}) > Q(\tau, \underline{x}), \forall \tau \in (0, t), \forall \underline{x} \in \Omega]$$

(Series formulation in both time and space coordinate)

Physics based modeling:  
 Total demand,  $Q(t) = D(t) + L(t) + \dots$   
 First Passage Time:  $T = \inf\{t : C(t) < Q(t), t \geq 0\}$

This made things a little more interesting that this example that we looked at it is quite instructive that we have a structure with more than one failure mode. So, in effect this has three elements one of flexure one of shear and one of excessive deflection. That is all fine but things become more interesting when we bring in or when we have to bring in some time dependence in one or more aspects of this problem.

So, here we see that there is some sort of time aspect both in the capacity which is degrading with time and in one or more here one load component. So, the live load is not invariant in time dead load is invariant in time but live load is not and here in the example you see that live load occurs as random pulses on the timeline. So, we have here a series formulation not only in critical locations but also in the time index.

So, for the structure to be safe for the structural system to be safe over the entire duration  $T$  over its entire domain  $\Omega$  the capacity at each time at each of the critical locations has to be greater than the demand for the entire set, so, for the entire set of locations and for the entire duration. So, this actually is what we have referred to as a physics-based modeling where the total demand is time dependent random.

So, random process in some sense and one of the things that would come out of this modelling is that the first time that such a failure happens. So, the first time at any location that at any critical

location the capacity is less than the demand placed on the structure at that location. So, that gives me the concept of a first passage. So, with all this discussion we are now able to list what makes structure reliability a bit different from what is often discussed in general reliability engineering.

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## Structural reliability – unique aspects

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- Mechanics based formulation (capacity-demand-time) of performance (safe vs. failed) is necessary
  - limit state functions must be defined
  - Significant uncertainties exist in mathematical models
  - Distributed system – many failure modes, many dofs
- Time to failure is not directly available
- Non-repairable in nature

- Active redundancy
  - load sharing
- Load path dependence
  - Different loading vs unloading behaviour
- Load redistribution after initial/successive member failure
- Presence of dominant sequences within the same cut set
- Presence of non-linearities
- Time dependence of load and strength properties
- Possible statistical dependence among system properties



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So, first of all our formulations are almost always mechanics based which is known as capacity demand or capacity demand time type formulation uh. So, that is what we call a limit state limitator for performance there could be uncertainties in the mathematical models the systems are distributed in nature and there could be there are generally more than one failure modes many degrees of freedom to describe my system.

So, all of these causes the time to failure not to be directly available not to be directly measurable, so, we can only as we saw in the previous example involving the cantilever beam that it can only be measured indirectly once we know the mechanics of the problem and the time properties of the loads from the systems perspective system modeling perspective it is a non-repairable system.

So, the structure cannot be taken offline and replaced by an identical new system it can be maintained but it cannot be replaced. So, non-repairable in technical language it has active redundancy. So, all the elements are loaded and there is load sharing. So, that brings

independence there is load path dependence. So, an example of that would be the loading and unloading behaviour could be different there could be load reversals load redistribution after some initial or a sequence of number failures.

There could be dominant sequences in the same cut set which means that it could be that members 1, 2 and 3 form a minimal cut set but one of those is most likely to fail first than the other. So, if we want to find the sequence probability or that cut set probability we need to be able to look at the correct sequence otherwise we might underestimate the that minimal cut set probability. There are non-linearity's both in material behaviour and in geometric aspects of the problem.

So, that that brings in So, we cannot use simple proportional behaviour that brings in some more challenges and then as we saw here that load and strength properties can change with time randomly and finally there could be dependence statistical dependence among the system properties at different locations are distributed throughout the system. So, all of this makes structure reliability more interesting sometimes more challenging to solve.

So, this is going to be our plan our plan for the rest of this course how we are going to approach structured library. So, it is a step by step plan.

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## Structural reliability – step by step plan

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1. Time dependent component reliability – phenomenological approach ⇒ Random TTF
2. Time dependent system reliability - phenomenological approach ⇒ Random TTF
3. Physics based component reliability formulation – time-invariant case ⇒ Random C and D; time dependent case ⇒ first passage problem
4. Physics based system reliability formulation – time-invariant case ⇒ Boolean combination of component limit states

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First we are going to look at starting with today's lecture time dependent reliability from a completely phenomenological approach for a component. So, our focus will be the random TTF time to failure. Then the next step would be time dependent system reliability continuing with the TTF approach. So, from element we move to system the third step would be the physics-based component reliability formulation.

First we will look at the time invariant case and then the time dependent case and do some first passage formulation in the end. So, in step three we would be graduating from a phenomenological time to failure based modeling to a physics-based capacity demand modelling and finally we would look at systems reliability with the same sort of Boolean combination of elements into system description that we have already looked at.

So, that would be the plan and that brings us to today's topic the time to failure component based description of element reliability.