

**Structural Reliability**  
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**Lecture –108**  
**Representation of Systems (Part -12)**

(Refer Slide Time: 00:27)

### System representation – minimal cut sets

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Recall:  
 $X_i = \text{indicator fn for element } i = \begin{cases} 1 & \text{if element is working} \\ 0 & \text{if element not working} \end{cases}$

System structure function:  
 $X_{sys} = \alpha(X) = \begin{cases} 1 & \text{if system is up} \\ 0 & \text{if system is down} \end{cases}$

Structure function of minimal cut set  $C_i$ :

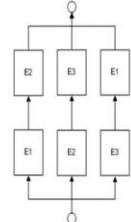
$$X_{C_i} = 1 - \prod_{j=1}^{k_i} (1 - X_{j_i})$$

Structure function of system:

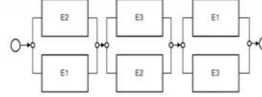
$$X_{sys} = \prod_{i=1}^k X_{C_i}$$

Recall, for a "2 out of 3 system":  
 $X_{sys} = 1 - (1 - X_1 X_2)(1 - X_2 X_3)(1 - X_1 X_3)$   
 $= X_1 X_2 + X_2 X_3 + X_1 X_3 - 2X_1 X_2 X_3$


Where did the higher order terms go? Binary variables:  
 $X_i = X_i^2 = X_i^3$  etc.



3 minimal cut sets:  
 $\{1,2\}, \{2,3\}, \{1,3\}$



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 Lecture 13  
 Representation  
 of systems



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The minimal cut sets let us write out the system structure function in a very intuitive manner. Recall that  $X$  is the indicator function for an element and the system structure function relates the system outcome one or zero in terms of the element outcomes. So, we would like to express the system structure function in terms of the minimal cut sets that we identify for the system.

So, we have looked at this what the structure function for a purely parallel system would look like and since the minimal cut sets in themselves are purely parallel. So, the structure function for each such cut set for the  $i^{\text{th}}$  cut set  $C_i$  would look like this as you see on your screen in terms of the indicator function for all the constituent elements of that cut set. And also the cut sets themselves the minimal cut sets themselves are arranged in series.

So, the final system structure function would look like as we have already derived the product of the individual minimal cut set structure function. So, this would be actually a very convenient

way of writing out the system structure function for any system that we have. So, let us look at the first such system that we considered a few slides back it was a 2 out of 3 system and you remember that we wrote out or we claimed that this is how the structure function looks like.

On the top right you see the block diagram for this system. Now since this is a 2 out of 3 system we can write out the final answer in terms of the expression that you see in the second line and we will see that we will try to arrive at the same answer through the approach that we have put in the box in the middle of your screen. Now you might be wondering as to how the two equations that you see are equal.

Because in the first line we have we should have 6th order quantities terms and whereas here the highest order that we have is three. So, what happened to the higher order terms it is not a mystery if you want to work it out please do. So, otherwise the answer is actually very simple it is that these all these element state functions the indicator functions they are binary in nature. So,  $X_i$  and  $X_i^2$  and  $X_i^3$  they are all the same because it is either 0 or 1.

So that is how we can get rid of the higher order terms. Now in terms of this system there are 3 minimal cut sets 1, 2; 2, 3 and 1, 3. So, we can write an equivalent block diagram by putting these cut sets in series. So, these two block diagrams are completely equivalent and if you write out the each cut sets structure function and then multiply them together you will arrive at the system structure function that you see in the second line which is  $X_1 X_2 + X_2 X_3 + X_1 X_3 - 2 X_1 X_2 X_3$ .