

**Structural Reliability**  
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**Lecture –107**  
**Representation of Systems (Part -11)**

After looking at reliability block diagrams in this lecture for system representation we are going to look at a different method that of cut sets and particularly minimal cut sets. The block diagrams were a success oriented approach to represent the system and cut sets is are kind of failure oriented in the sense that we try to identify all those elements that must fail in order for the system to fail.

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## System representation – Cut sets

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- A set of elements of a system is a *cut set* if the failure of all members of the cut set causes system failure.
  - Hence, a cut set is a parallel arrangement of its members
  - the cut sets of a system are arranged in series.
- A *minimal cut set* is one that if any element is removed from it, the subset no longer remains a cut set.
- Each minimal cut set behaves like a purely parallel system
- Even if individual element failures are independent, the cut sets are generally not independent owing to the presence of the same component(s) in more than one cut sets.

System failure is the union of all cut sets (not necessarily minimal cut sets):

$$F_{sys} = \bigcup_{\substack{\text{all cut sets} \\ \text{minimal or not}}} C_i$$

A minimal cut set,  $C_i$ , is the intersection of all its constituent failure events:


$$C_i = F_{i_1} F_{i_2} \dots F_{i_m}$$

$F_{i_j}$  = failure of  $E_{i_j}$ ,  
 $E_{i_j}$  = the  $j^{\text{th}}$  member of the  $i^{\text{th}}$  minimal cut set  
 $X_{i_j}$  = state of  $E_{i_j}$

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So, let us define them a set of elements is a cut set if the failure of all members of that cut set causes system failure. Now a cut set therefore is a parallel arrangement of its members and these cut sets arranged in series. So, you see the system failure event is the union of all the cut sets. Now a minimal cut set is a very useful and efficient way of looking at cut sets it is a cut set if it loses any of its members and it does not remain a cut set is a minimal cut set.

So, we try to identify minimal cut sets in all situations and work with them. So, let us define  $C_i$  the  $i^{\text{th}}$  minimal cut set in terms of its constituents. So, say cut set  $i$  has  $m$  members and they

are  $F_{i1}$  and  $F_{i2}$  all the way up to  $F_{in}$ . So, they are all arranged in parallel and we if we want to work with them further we can associate the state of the element  $E_{ij}$  for each member of cut set  $i$ .

Each minimal cut set behaves like a purely parallel system and typically elements are repeated across cut sets. So, even if the elements are independent because of the same element showing up in more than one cut set the minimal cut sets are not necessarily mutually independent. So, that needs to be kept in mind.

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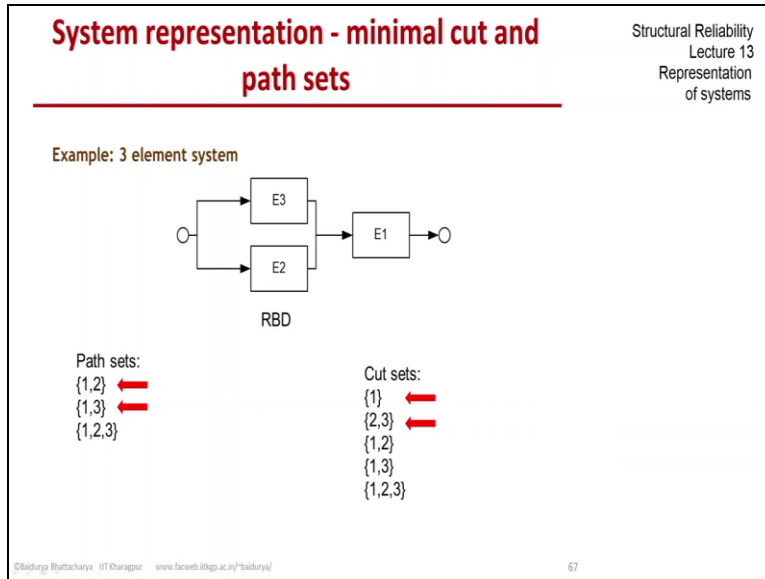


Now along with the cut set there is a dual way of defining of representing systems and that is through path sets. So, if we just switch the logic a set of elements is a path set if the survival of all members of that path sets imply system survival and just like the cut sets where a parallel arrangement of its members a path set is a series arrangement and the path sets themselves are arranged in parallel.

And just like we had the minimal cut set we also have the minimal path set which as you have guessed if any member is removed from the path set and it stops being a path set then it is a minimal path set to begin with. In structural reliability we do not really work. So, much with path sets it is much more intuitive to look at cut sets and minimal cut sets. So, we are not going to focus any more on the path set approach.

Although in today's example there will be one or two cases where I will also discuss them starting with the next.

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So, let us look at this very simple three element system what you see is the reliability block diagram and let us find out its path sets and cut sets and identify those that are minimal. So, it is quite straightforward for this simple system that element one must work for the system to work and at least one of the elements two and three have to work in order for the system to work. So, the path sets of the system are 1, 2.

The paths the next one is 1, 3 and we could also have the entire set 1, 2, 3. Now I am sure you have been able to guess which ones are the minimal path sets the first one because if you remove any one of them it no longer remains a path set, the second one is also a minimal path set. So, these first two that I have marked with the red arrows are the minimal path sets but the third one is not a minimal path set. We can also identify the cut sets and it is not necessary that the number of patches the number of cut sets have to be the same.

So, the first cut set is 1, obviously if one fails then the system fails 2, 3 would also be another cut set 1, 2 would be the third cut set we could have more we could have 1, 3 and we could have 1, 2 and 3 and now which one of these are minimal cut sets obviously 1 is a minimal cut set 2, 3 is

also a minimal cut set because if we remove either one of them 2 or 3 then it no longer remains a cut set. And these 2 are the only minimal cut sets all the others are not minimal.