

**Structural Reliability**  
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**Lecture –106**  
**Representation of Systems (Part -10)**

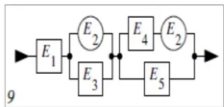
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### System representation – Reliability block diagram

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Examples: RBD with repeated element



Element E2 appears twice in the RBD. Not in the hardware!

$$R_5 = R_2 R_1 (R_4 + R_5 - R_4 R_5) + (1 - R_2) R_1 R_3 R_5$$



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A similar approach can be taken when the system logic is a little more complicated as we see here the element E2 it appears twice in the block diagram but obviously it is not twice present physically in the hardware. So, this is one way of showing the role of the element E2 in the system logic. So, we would proceed similarly as we did in the case of the bridge systems if you want to work through this please go ahead otherwise let me present the solution. It would again giving you the hint involve conditioning on the element E2.

So, in one case it is working for sure and the other part of the partition would be that it has failed for sure. So, this would give you two simpler configurations again involving the elements that are mutually independent 1, 3, 4 and 5. So, it would be straightforward to solve those two simpler systems and this would be the answer.

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## System representation – 2 out of 3 redundancy example

Recall:

$$X_i = \text{indicator fn for element } i = \begin{cases} 1 & \text{if element is working} \\ 0 & \text{if element not working} \end{cases}$$

System structure function:

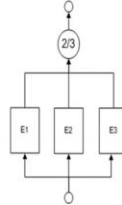
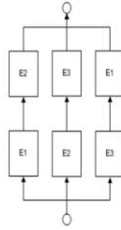
$$X_{sys} = \alpha(\underline{X}) = \begin{cases} 1 & \text{if system is up} \\ 0 & \text{if system is down} \end{cases}$$

Given a 3 element system.

System works as long as at least 2 elements work  
("2 out of 3 system")

$$X_{sys} = \begin{cases} 1 & \text{if } \sum_{i=1}^3 X_i \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$X_{sys} = 1 - (1 - X_1)(1 - X_2)(1 - X_3)$$



We now see how to handle situations where the same element occurs repeatedly in the reliability block diagram because of its role in system failure or system survival we just saw one. But let us take up one example from structural engineering because these things occur frequently in structures. So, we start with an example for what is known as a two out of three redundant system. We are again going to look at this formally later but this is basically what it means recall that the indicator function of element  $i$  is  $X_i$ .

So, it is 1 if the element is working and 0 otherwise. So, we have the system's structure function  $\alpha$  in terms of the element states now if we are given a 3 element system and the system works as long as at least two elements work that is very important at least two elements work in this which is that's why it is called two out of three systems or in general  $m$  out of  $n$  systems. So the structure function can be given simply as one if the sum of all the  $X$ 's is at least two or zero otherwise.

Or if you want an analytical expression it is quite clever it is the expression that you see on the bottom of your screen what would be the reliability block diagram of such a system. So, it could if you want to go through all the possible combinations this is what it would look like. So, since at least two have to work it is  $E_1, E_2$  have to be in series or  $E_2, E_3$  have to be in series or  $E_3, E_1$  have to be in series and this obviously includes the case when all three of them are working as well.

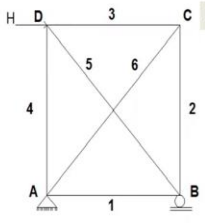
There is a more concise way of expressing this and that would be we just put a block there called two out of three and we pass E1, E2 and E3 through them.

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### System representation – Reliability block diagram

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- Indeterminate truss
- Brittle failure



- Draw the RBD of the truss

**Structural analysis**     EA=1, L=1

**KGlobal=**

1.3536	0.3536	-1.0000	0	-0.3536	-0.3536	0	0
0.3536	1.3536	0	0	-0.3536	-0.3536	0	-1.0000
-1.0000	0	1.3536	-0.3536	0	0	-0.3536	0.3536
0	0	-0.3536	1.3536	0	-1.0000	0.3536	-0.3536
-0.3536	-0.3536	0	0	1.3536	0.3536	-1.0000	0
-0.3536	-0.3536	0	-1.0000	0.3536	1.3536	0	0
0	0	-0.3536	0.3536	-1.0000	0	1.3536	-0.3536
0	-1.0000	0.3536	-0.3536	0	0	-0.3536	1.3536

**DOFs = 1\* 2\* 3 4\* 5 6 7 8**     Rank = 5  
 \* = restricted (to zero)

**Forces = \* \* 0 \* 0 0 1 0**  
 \* = reactions (unknown)

**Kreduced=**

1.3536	0	0	-0.3536	0.3536
0	1.3536	0.3536	-1.0000	0
0	0.3536	1.3536	0	0
-0.3536	-1.0000	0	1.3536	-0.3536
0.3536	0	0	-0.3536	1.3536

**Rank =**

Solve: Ureduced = Kreduced %(-1)    Freduced = 0.5 1.91 -0.5 2.41 0

So, this is the truss that we would like to analyze this has 6 elements two diagonals elements five and six simply supported at A and B, A is a hinged support and B is a roller support uh. So, we need to draw the RBD of the truss again depending on our understanding of the structure and how its boundary conditions are what its material is and. So, on we can find its reliability block diagram by inspection or we might have to do some structural analysis as well.

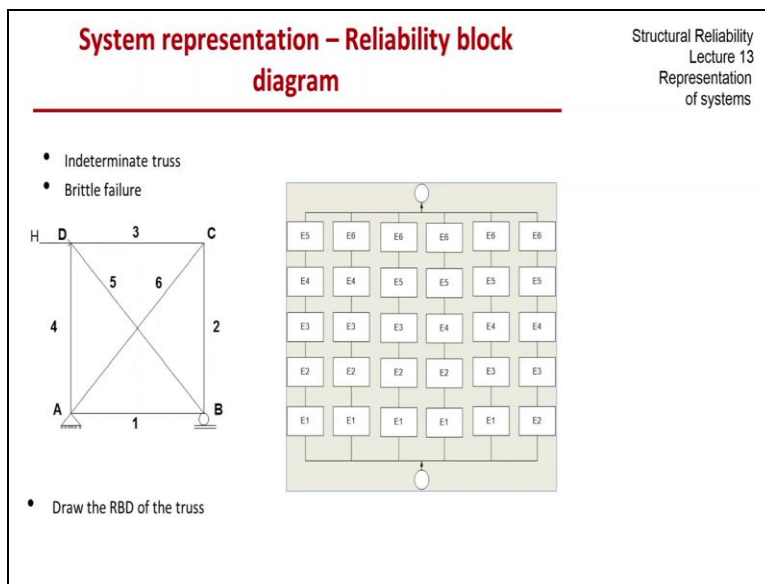
So, here by inspection we know that this is one degree redundant but suppose we wanted to verify this by analyzing the structure and let us say as we have said here it is elastic brittle. So, we could take up the stiffness method of analysis there are eight degrees of freedom in all to each at a b c and d and obviously we need to restrict some of them and because of the boundary conditions here at A and B.

So, we would restrict three of them the rank of the global stiffness matrix is 5. So, we remove 3 degrees of freedom and we have the reduced stiffness matrix and we can invert that and solve the structure. So, the point here is that the structure is stable when we have this configuration. There are various ways in which we can as we remove members as the structure gets damaged there are

various ways of finding if the structure has collapsed has become unstable or not.

We **we** can just for this simple example we can see when the stiffness matrix no longer remains invertible. So, let us **let us** remove one member say let us remove number one and we can see that the structure is still stable and we can invert this and we can get answers. So, if member 1 is removed the structure still has not failed which is which confirms our understanding of the degree of static indeterminacy for this structure if we remove one more member the structure no longer remains stable.

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So, the structure has failed. So, if we do this again and again for various combinations pairwise for lost elements we will see that this any 5 have to work any five of these six elements have to work. So, we could draw an RBD and say that you know there are six elements E1 through E6 and five out of six configuration which when expanded looks like this there are 6 ways of choosing five elements from the group of six.

So, this would be the expanded form of the reliability block diagram of the square truss in question that has two diagonal values.