Structural Reliability

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Lecture –101
Representation of Systems (Part -05)

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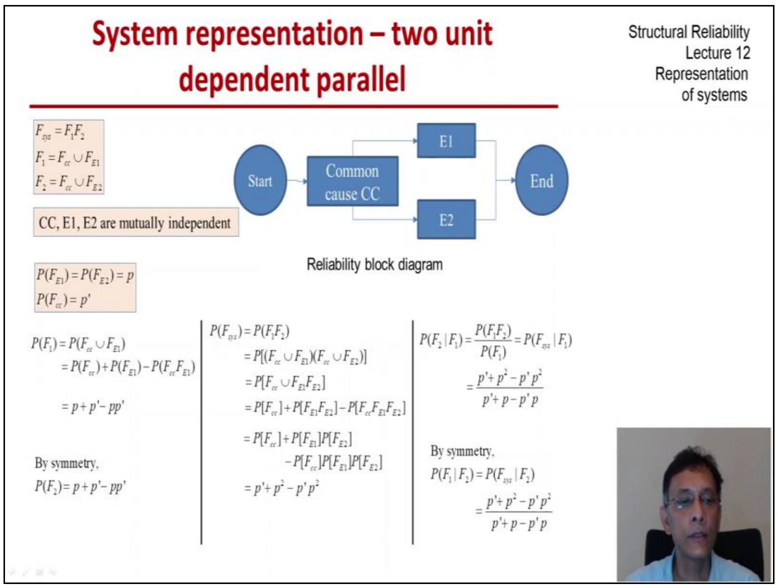

We end this lecture with looking at a two unit parallel system but with dependence between the two units and thereby bring out some interesting observations.. So, the two units are in parallel. So, the system failure event is still F 1 intersection F 2 but the reason that the two units are dependent upon and F 2 are dependent is because of the presence of a common cause. So, F 1 is F cc which is common cause union F 1 and F 2 is F cc mean F E2.

So, what are E1 and E2 are events that constitute part of the unit 1 and E2 for unit 2 the common cause E1 and E2 are all mutually independent but because CC shows up in both one and two it causes dependence between the failure events F 1 and F 2. Let me show you the reliability block diagram we have not introduced it but it is quite intuitive. So, it clearly shows how the events are connected E1 and E2 are in parallel but common cause is in series with them.

And so, E1 and CC together constitute F 1 and E2 and CC together constitute F 2 and thereby F

1 and F 2 are debugged. So, we want to find out the failure events of F 1 its probability, probability of F 2 and the system failure probability as well as the conditional failure probabilities in terms of the relative values of the common cause in the probability and the E 1 E 2 probabilities. We take the simplest assumption that the problem is symmetric in F 1 and F 2.

So, E1 and E2 are identical ah. So, P of F E 1 and P of F E 2 each is equal to p and the common cause in the probability is p prime and let us now derive the P of F1 and P of F 6 and so on in terms of this P and P prime. So, P of F 1 by definition is P of F cc union P of F P 1 and if we expand it we come up with the usual P of A minor + P of P - P of AB and by putting in the values because now they are independent F cc and F E1 are independent.

So, it is simply the P of F 1 is P + P prime - P P prime because of symmetry P of F 2 would have an identical value. So, we have derived the probability of individual element failures in terms of the common cause failure probability. The next would be to derive the system failure probabilities. So, which by definition is F 1 F 2. So, if we expand F 1 in terms of the common cause and F 3 in terms of the common cause then we can work through the steps P of F cc is P of F common cause union P union F E1 F E2.

And then by expanding that and bringing in the fact that they are all independent now we can express the system failure probability as in terms of P prime and P over 2. So, we have derived P of x is now for the sake of completeness let us also find out the measure of dependence because F 1 and F 2 are not dependent they are related to the common cause. So, let us find out P of F 2 given F 1 it also turns out that because of the nature of the system p of F 2 given F 1 is also P of F sys is F 1.

So, by definition P of F 2 given F 1 is P upon F 2 over P of F 1 and we have already derived each of those. So, that would be P prime + P squared - P prime P squared over P prime + P - P prime p. By symmetry P of F 2 given F 1 is also the same as P of F 1 given F 2. Now let us see how the common cause failure probability p prime affects the entire picture uh.

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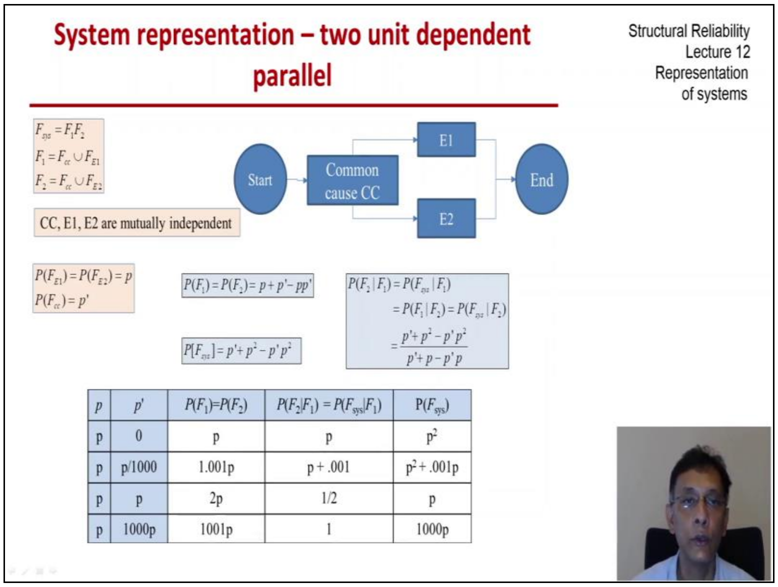

So, let us go to the next slide and this is what we have we have found out P of F 1 and F 2 individually we have found out the system failure probability. We have also found out the conditional failure probabilities. Now let us put four possible values of P prime in relation to P uh. So, the common cause failure probability; in the ideal case in the case that we assume that F 1 and F 2 are independent. So, P prime is 0. So, there is no common cause then this very small P prime which is 1000th of P.

So, only 0.001 Pp is there as the common cause failure probability the third possibility is that P prime and P are equal and then P prime is huge. So, the common cause failure probability lead it is it is the dominant failure probability. So, what happens to P of F 1 or P of F 2. So, when there is no common cause then we go back to the classical case. So, P of F 1 is P when there is a slight common cause probability.

So, it is 10 to the power -3 times P so 1000th of the independent event probabilities then P of F 1 or F 2 is just slightly larger. So, it is 1.001 P when the two are equal we have twice P. So, the probability of one and two they become double and then when the common cause dominates then it is obviously that. So, 1001 times P. But what happens to the system failure probability and what happens to the conditional failure probability.

Let us take a look at that and that will be very instructive. So, when P prime is 0 then the system failure probability as we know is P squared but when P prime is as little as 0.001 P which is the second row then we see a huge increase in the system failure probability. It is P squared plus an addition of 0.001 P. So, if P is small I say P is the order of 10 to the -4 or 5 or 6 then that second term that 0.001 P starts to dominate.

So, even a very small common cause is going to have an overwhelming increase in the system failure gravity and that is the same thing which happens in the conditional failure probability when they were independent F 2 given F 1 was still P but when there is a slight common cause then it goes up by 0.001 and if we in the order of -6 then obviously you can see that the common cause starts to dominate.

When P and P prime are equal then the system loses all benefit of being in parallel. So, the probability of system failure is now equal to P instead of P squared the conditional failure probability of F 2 given F 1 is half as much as half and when in the last row P prime starts to dominate then obviously P of F 2 given F 1 is practically 1. So if one fails the other is almost certain to fail and the system failure probability is enormous uh. So, it is one thousand times.

Obviously these are their approximations in these expressions are they continue to be small enough uh. So, but the idea is there is a potentially huge contribution to common cause failure when two units are in parallel but they are not independent.