Structural Reliability
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Lecture –100
Representation of Systems (Part -04)

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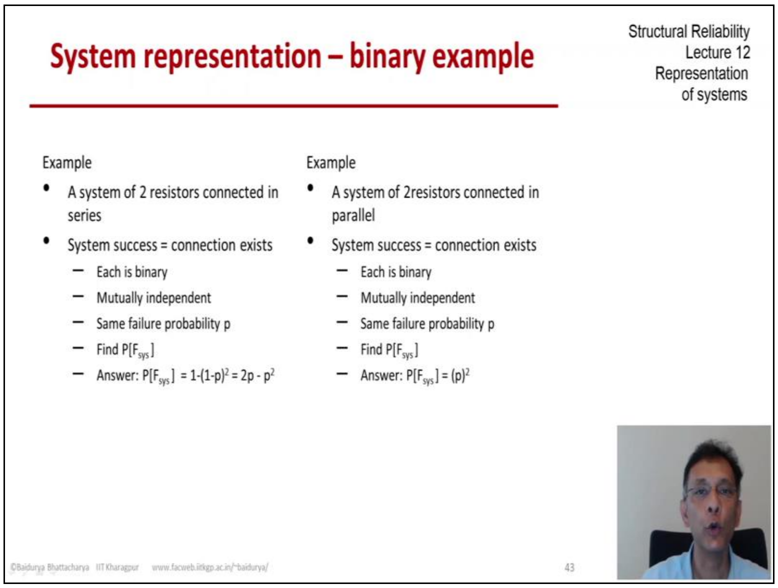

Let us solve some problems involving a system composed of just two elements. The simplest we have already looked at in various situations and one example is here on the screen that a system consists of two binary units, two resistors connected in series and the system success is defined as connection exists across these two each of these resistors is binary in nature they are mutually independent.

They have the same failure probability P and we have solved for the system failure probability for such a simple system several times in the past. So, the answer is the system failure probability is twice p - p square because F 6 is F 1 union F 2. So, P of F 6 1 + PF 2 - PF 1 F 2 and because they're independent it is p + p - p squared we need to remember this because we are going to look at a little more complicated problem a three state problem and the question will be is it going to look the same.

So, this 2p - p squared we are going to come back to. The parallel configuration involving these two resistors would have their own picture. So, we have also looked at these under various guises. The system success once again is that the connection exists across these two and the system for the probability is p square because system failure is intersection of F 1 and F 2 and because they are independent the probability is the product of the individual probabilities and this is also something we need to remember. So, on the left please remember 2p - p squared and on the right please remember p squared.

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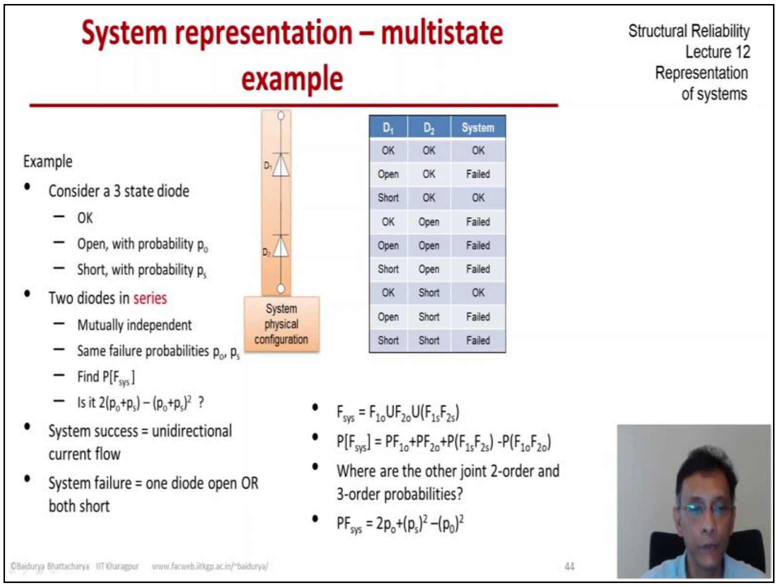

With that let us look at a three-state system. So, we have a three-state diode and the three states are like this the diode is okay. So, it is functioning the diode is open which means it is not conducting and the diode is short. The probability of being open is P o and parameter being short is P s. So, the probability of being okay is 1 - P o + P s. With this in mind we define a configuration that the two diodes are in series.

So, the question is and as before they are mutually independent both of them have the same failure probabilities P o and P s P open and is short the question is that what the system failure probability is. Please remember what we had before it is 2p - p squared in the case of the two registers. So, here in the three state system is it still twice p of failure which it would seem that you know because open is a failure state short as a failure state.

So, is it like going to be the same twice of P o + P s - P a + P s square. Let us work through the system but here we need to make sure that we factor in what the system success is. So, the system success is what a diode is supposed to do that there is unidirectional current flow. So, that is that is very important. Now if we now think of the logic of such a system. So, if system success is unidirectional current flow then system failure would be that either one diode is open or both have shorted.

If that happens then either there is no current flow or there is bi-directional current. So, if you like you can also type up the truth table the enumeration that we discussed earlier in this lecture. And here instead of 2 to the power of n we have 3 to the power of n. So, there are 9 states and I have enumerated all of them and you can see that you know this logic that we had on the left column that system failure is one diagonal or both short and system success is in the directional current flow that is captured completely in this enumeration.

So, now if you want to work through this problem please pause the video otherwise let me let me present the solutions. And the first step would be to write the system failure event in terms of the diode states of one and the other. So, that is D 1 and D 2. So, the system failure is F 1 open or F 2 open or both have shorted. So, that is the total description of the system failed.

Now we can find the probability of this event. So, it is basically a P of A union B union C. So, there are three sets are there and if we write through the inclusion exclusion formula there would be P A + P B + P C - P AB - P BC – P C and so on. So, the question is that why have I stopped just here where are the other second order and third order probabilities I would ask you to work those out but the hint is that those events are such that null the null set ends up being answered.

So, if you work through this the answer is twice P open + P short squared - P open square which is not which is definitely not what the case would be had we taken the simplistic view of the of the resistor situation. So, this is how the system state and the system logic needs to take into account how the elements behave and how they are put together and how the system function is in terms of the element functions.

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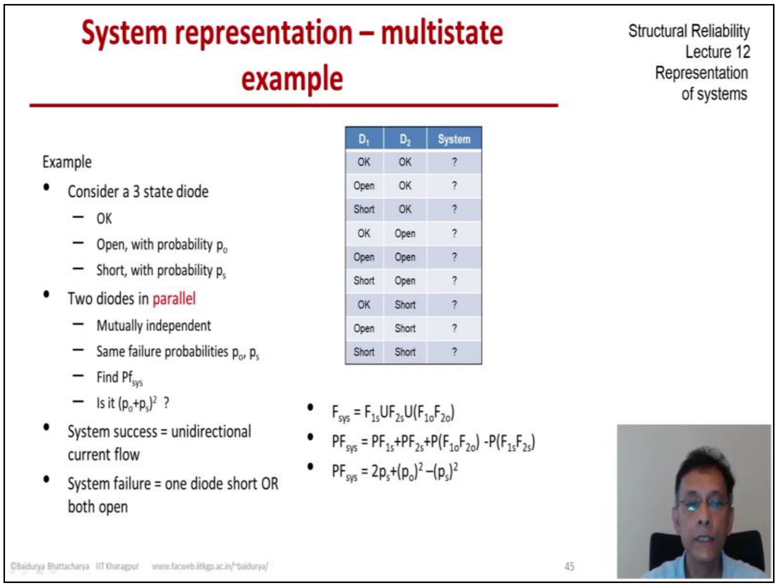

We could do the same thing by looking at the parallel configuration of these two three state diets. So, it is the same except that the two diodes are in parallel and if you remember we had P squared in the registers case uh. So, the question is that is the system failure probability now are the square of the sum of P open + P short and again it is the same logic that the system successes there is in the directional current flow and system failure is is either one that are shorter or both are open soon because they are in parallel.

And if you like you can fill up this truth table and if you want to work through this please pause the video otherwise let me present the answers uh for this situation. So, proceeding as before this is the system failure event and we have the system for the probability and this is the answer which is different from the square of P o + P s.