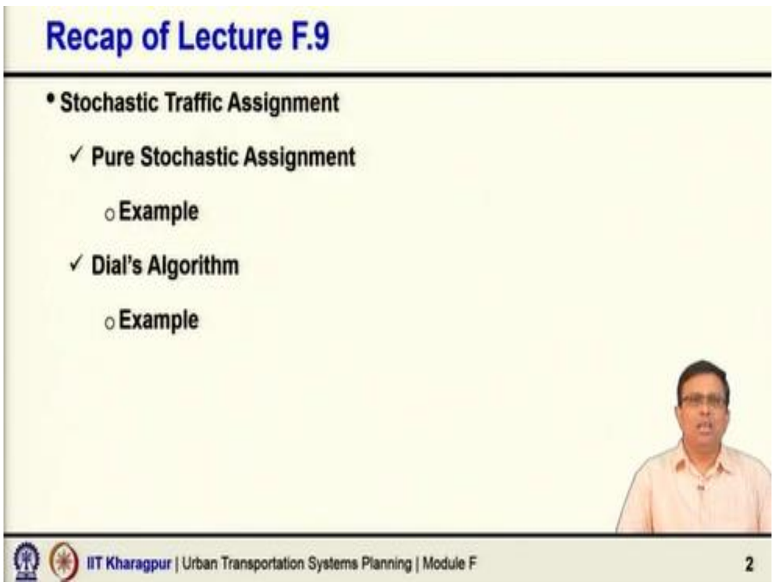
Urban Transportation Systems Planning
Prof. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 50
Stochastic Traffic Assignment II and Dynamic Traffic Assignment

Welcome to Module F lecture 10. In this lecture we shall discuss and rather we will continue our discussion about stochastic traffic assignment and also, before we close, we shall give you a brief introduction to dynamic traffic assignment technique.

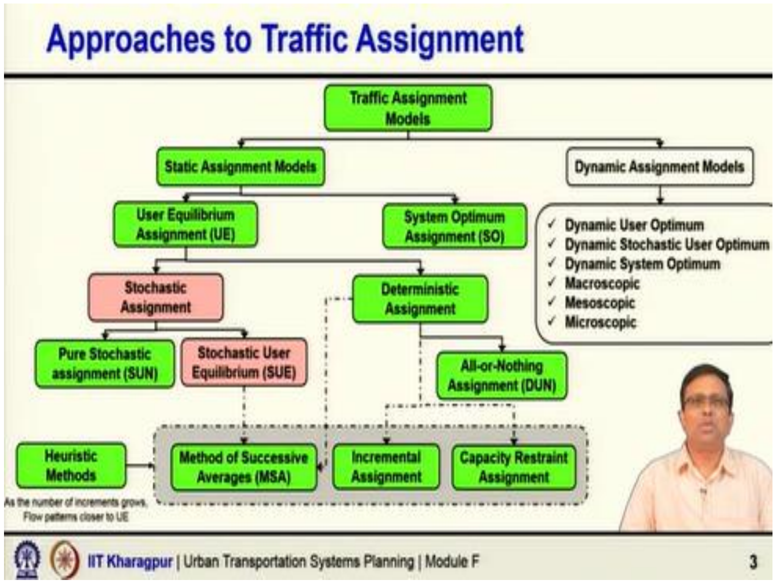
(Refer Slide Time: 00:32)



In lecture 9, the last lecture we discussed about stochastic traffic assignment. Why we need to consider this stochasticity? The perception error, the variability due to the perception, the variability in the perceptive travel time or travel cost and then use of multiple paths because of this variability in the perceived travel time. So, then we talked about the pure stochastic assignment also took the example problem and solved that using the Dial's Algorithm.

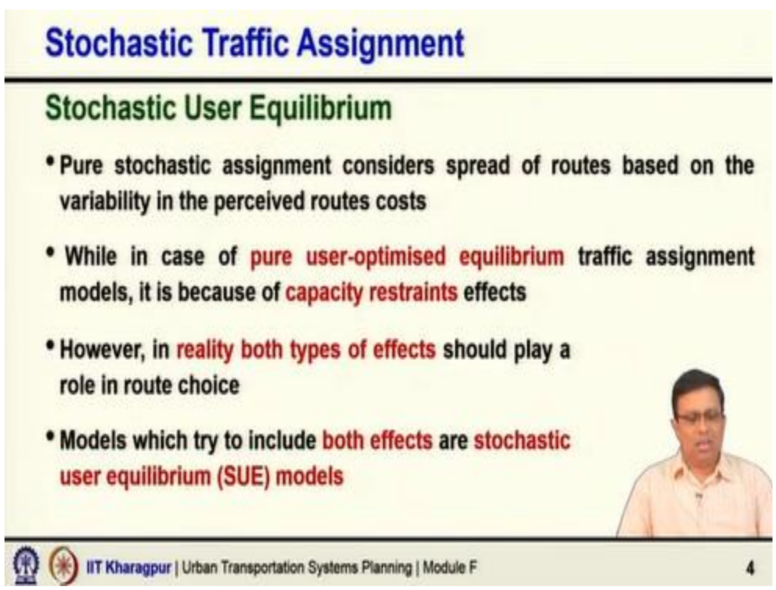
The forward pass and then the backward pass so, with an example, I have explained. So, now you know what is stochastic traffic assignment, why we go for it? And what is pure stochastic traffic assignment?

(Refer Slide Time: 01:38)



So, in continuation to that we shall discuss in this class or lecture about stochastic user equilibrium. So, it is a stochastic assignment we discussed used stochastic assignment and in this lecture, we shall discuss about stochastic user equilibrium SUE.

(Refer Slide Time: 02:04)



Now, pure stochastic assignment considers spread of routes based on variability in the perceived route cost or the path cost or the route travel time or path travel time. So, we know that true travel time is not known information availability is limited. So, the available information not 100% information is available to everybody and the true travel time is not known. So, we take route individual we take route based on our perceived travel time.

So, pure stochastic assignment considered the spread of route to account for this variability in the preceptor time. In case of pure user-optimized equilibrium, traffic assignment or equilibrium traffic assignment model, this variability happens because of the capacity restraint effect. We say that if we do not consider stochasticity that is also but consider a realistic situation consider a typical urban condition urban transport condition.

Nearly, every big city we have demand and supply there is great imbalance on many links or many segments, the demand may be even more than the available capacity during certain hours of the day and even when the demand is less than the capacity then also the demand is very significant. So, there will be congestion effect and because of that the travel time will no more be the shortest path will know where the shortest path once the flow or the demand is assigned and alternative paths will be used.

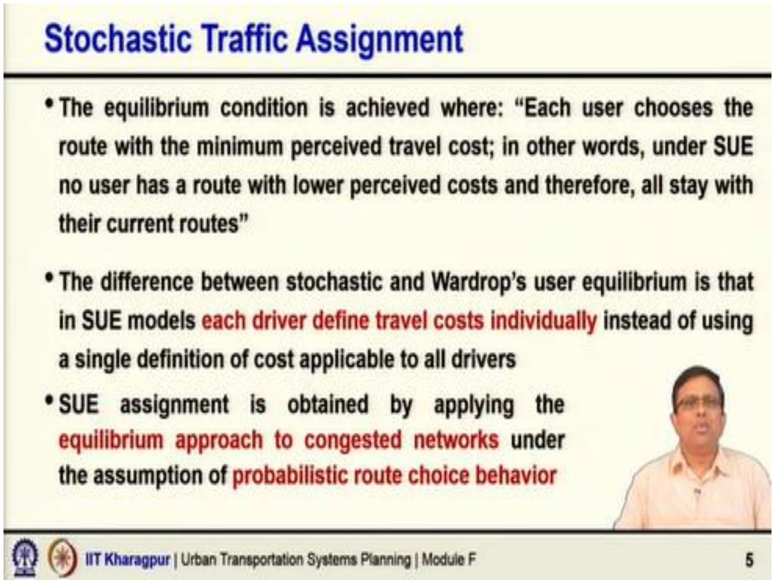
So, in pure user optimal equilibrium DUE, in pure user optimal equilibrium multiple paths are chosen because of the capacity consideration because of the congestion consideration. However, and so what is really happening? That in pure user optimal equilibrium multiple paths are taken because of the capacity consideration because of the congestion consideration and in pure stochastic assignment multiple paths are taken because of the variability in the perceived travel time or travel cost.

Now, what is bit more realistic is to consider both these aspects. So, in reality we consider a practical situation. This stochasticity also will be there; because people will not have all users will not have the true information about the travel time. So, there will be variability and everybody will actually try to optimize his or her perceived travel time rather than the actual travel time that is also true.

But, the other end is also true that roads will have capacity limitation demand may be more than the capacity or even when it is not more than the capacity then also there will be significant congestion effect and therefore because of that also multiple paths will be chosen. So, reality is a mixture of these two; stochasticity or the variability in the perceptive travel time and also the capacity constraint or capacity restraints and congestion effect.

So, when we try to consider both aspects together then the kind of equilibrium what we are saying is stochastic user equilibrium. So, not just stochastic assignment not just deterministic user equilibrium DUE, but it is stochastic user equilibrium. So, stochastic is also considered, capacitive restraint is also considered both are considered.

(Refer Slide Time: 06:56)



So, the equilibrium is achieved here with this kind of assumptions or reality. Each user chooses the route with the minimum perceived travel cost remember that, we are sticking to perceive travel cost. So, the variability is there stochasticity is there. So, each user chooses the road with the minimum perceived travel cost. In other words, under stochastic user equilibrium slight deviation from the deterministic user equilibrium carefully note this.

No user has a route with lower perceived cost, lower perceived cost and therefore, all stay with their current routes, in DUE deterministic user equilibrium perfect knowledge, no variability in the percept cost. So, all the routes will have equal travel time and we said any unused route will have either equal or higher travel time that was the assumption. Here we are deviating because we are everything is going around perceived travel time or perceived travel cost.

So, we are saying we are still reaching to an equilibrium where no user has a route with lower perceived cost. So, nobody is interested to change route from one route to another route simply because no individual is able to bring down the perceived cost by changing route. Everybody is not thinking still the same time that variability exists but whatever I think whatever you think whatever every other individual is thinking they are perceived travel time may be different for different paths.

But everybody is thinking that if I change route individually, I cannot bring down my perceive travel time further. So, with the variability in that perceived travel time, still we are reaching to an equilibrium. So, this equilibrium is going to be different then the deterministic user equilibrium, and because the perceptual times interval caused along different paths are different to different individuals. So, all the routes still will not have equal cost or equal time.

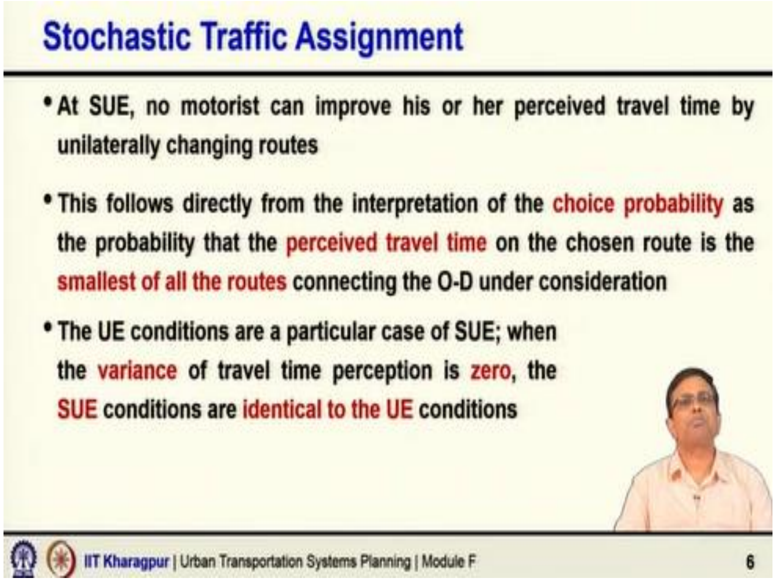
So, i to j may be there are four alternative paths, but all paths still may not give me and will not give me rather equal travel time. So, it is not going to be like this deterministic user equilibrium. But still the equilibrium will reach, why? Because everybody will feel like I cannot reduce my perceptive travel time further by changing the route and every individual will feel like that. So, that is the equilibrium.

What is equilibrium here? Nobody went nobody is interested to change the path further that is our equilibrium. So, the equilibrium will be when no one is changing the route further. So, flow is not changing further, but the travel time on all the alternative paths still may not be same. So, the difference between stochastic and Wardrop user equilibrium, as I said is that in SUE model each driver defined travel cost individually.

Please carefully note this word individually instead of using a single definition of cost and value applicable to all drivers. So, in SUE assignment is obtained by applying the equilibrium approach to congested networks under the assumption of probabilistic route choice behaviour. Stochasticity is there, so the model is a probabilistic model. So, route charge behaviour is the probabilistic like the mode choice behaviour.

Probabilistic mode choice model, similarly here also it will be probabilistic route choice model but then still the equilibrium will reach equilibrium will reach in a slightly different manner as I explain.

(Refer Slide Time: 12:17)



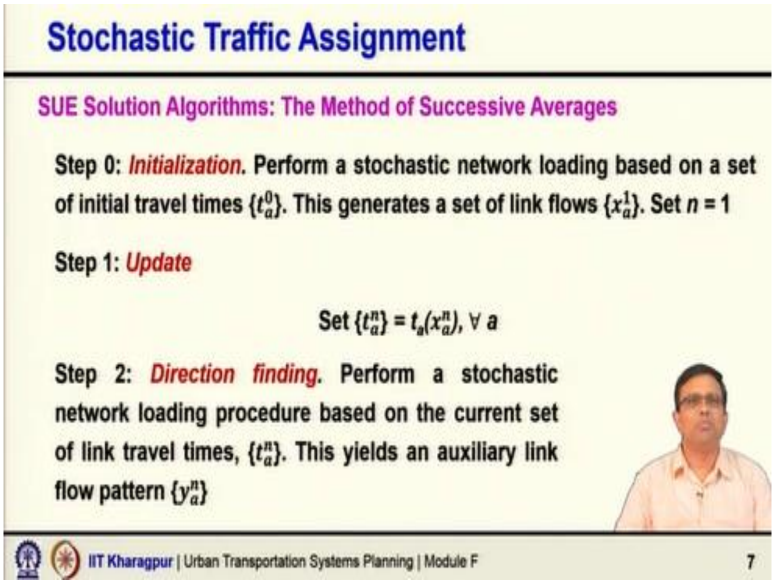
At SUE, no motorist can improve again I am repeating his or her perceived travel time no one can improve a reduced in this case his or her perceived travel time by unilaterally changing routes. So, this follows directly from the interpretation of the choice probability as the probability that the perceived travel time on the routes is the smallest on all routes connecting to the O-D under consideration, please carefully observe this statement.

What we are saying this follows directly from the interpretation of the choice probability as the probability that the precept travel time on the chosen route is the smallest of all the routes connecting the O-D under consideration. So, that is what the user stochastic user equilibrium is reached. The UE conditions are particular case of SUE, why I said so very simple. Why SUE? SUE because of the variability, that is what is the difference.

So, if you consider there is no variability or the variability is zero that varies down to UE or DUE deterministic under equilibrium. Generally, I am speaking UE. So, when stochasticity and without stochasticity, so without stochasticity is a specific case of stochasticity when the variability's zero. So, the UE conditions are a particular case of SUE when the variances are variation of travel time perception is zero.

The SUE conditions are identical to the UE condition you may call it DUE you may call it with capacity consideration or capacitive resistant consideration; you can call it as DUE.

(Refer Slide Time: 14:39)



Now, how to go ahead with this? So, we initialize do the initialization. Performing a stochastic network loading as we have discussed in lecture 9 previous lecture only based on a set of initial travel time. We know the initial travel time zero flow travel time is known and this generates the set of link flow, x a 1 set n = 1, n is the number of iterations, then what we are doing? We now know that how much flow has been assigned initially in the initialization part.

So, based on that, what is the travel time? With that flow we update then the travel time. Then we go for direction finding, direction finding means in this case perform a stochastic network loading procedure based on the current set of link travel time. So, we try to reassign with the current flow, current flow we reassign. But then this will yield what? This is this will yield and auxiliary link flow pattern.

Do you remember this auxiliary link flow? We discussed earlier in the context of deterministic user equilibrium assignment under method of successive average, yes that emissive technique. So, we are exactly now we initially do a stochastic network loading and then we further update the flow, do again the stochastic network loading but then whatever we are going to get we get that as auxiliary link flow pattern.

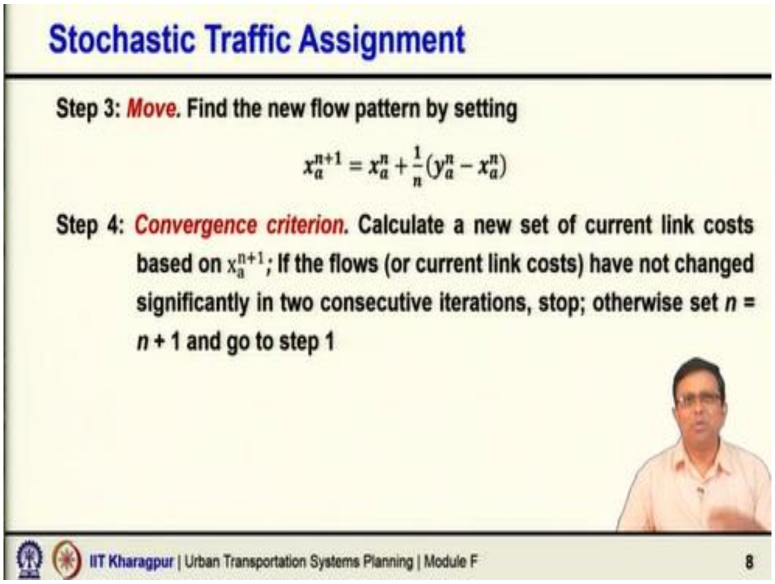
Exactly, the like the MSA what we have what you have already studied you already know we discussed all this with an example also. So, this auxiliary flow we shall get then we have to update you remember that we every time we update the link cost, we assign the flow again and there we did deterministic assignment. So, that means whatever is the shortest path along the new shortest path, we loaded entire flow was loaded along the shortest path to get the auxiliary link flow pattern.

But here the only difference is we shall again do a stochastic network loading. So, not everything will be loaded along the shortest path, but all paths will get their share as per the travel time and the link cost function based on the current loading that is the only difference but otherwise this method of successive average is absolutely similar no difference. Only the auxiliary flow how you got there we did deterministic assignments.

So, we with the current loading we got the revised travel time and accordingly based on this revised travel time we then again assigned the flow. But the complete flow was assigned along the shortest path and multiple times we did that, we updated the link flow and again and again started calculating recalculating the travel time and accordingly load the network but every time we are loading, we are loading using along the shortest path.

The current shortest path just assigns every flow there but in this case when you are using MSA under SUVs stochastic user equilibrium, then every time the assignment is done this is stochastic network loading. So, stochastic assignment is done, that is the only difference, the remaining is simple as it was stated earlier. You get the auxiliary flow then you remember this equation.

(Refer Slide Time: 19:05)

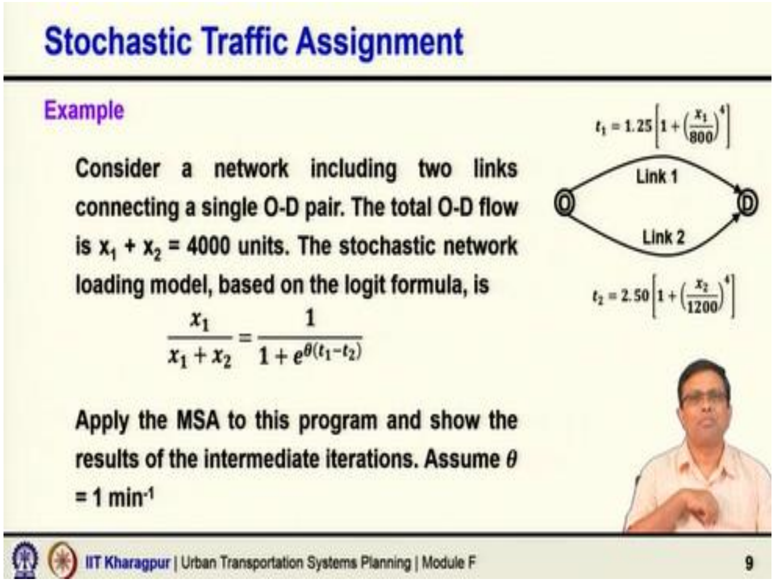


Now, we actually update the link flow that was the auxiliary flow actually link flow how we update x a n + 1 = x, x a n + 1 = x a n, whatever was the previous flow plus 1 by n into within bracket, y a n - x a n. What is x a n? x a n the previous whatever was the flow and y a n is what y a n is the auxiliary floor, what you have got in the nth iteration. So, accordingly we update the link flows for n + 1 nth iteration.

And then in this case, what is the convergence criteria? Earlier, when we applied this MSA, what was our convergence criteria? Convergence criteria was when the travel time on alternative paths are same. Here no; here we do not use travel time because still travel time variations will be there because it is stochastic assignment fundamentally. So, what we do? We see when iteration to iteration the flows along the alternative paths are not changing significantly.

So, you set a limit what is acceptable error and when the change in path flow is less than that maximum change in the path flow is less than your permissible limit then you say I have reached an equilibrium condition, and that is the answer. And again, under that condition also the travel time on all alternative paths are not going to be same because it is stochastic assignment.

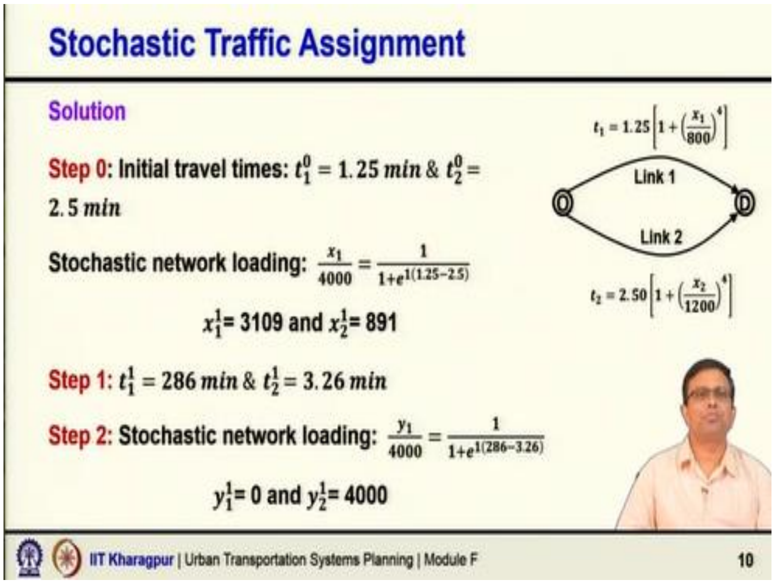
(Refer Slide Time: 20:59)



So, they can example here; there are two links, link 1 link 2. The travel time functions are given t 1 = 1.25 + 1.25 within bracket 1 + x 1 by 800 to the power 4 just like PPR function. So, link 2 also the travel time function is given. We are saying that 4000 unit flows are to be distributed on link 1 and link 2 and this is the Logit formula that may be applied 1 by 1 + n to the power theta into t 1 - t 2.

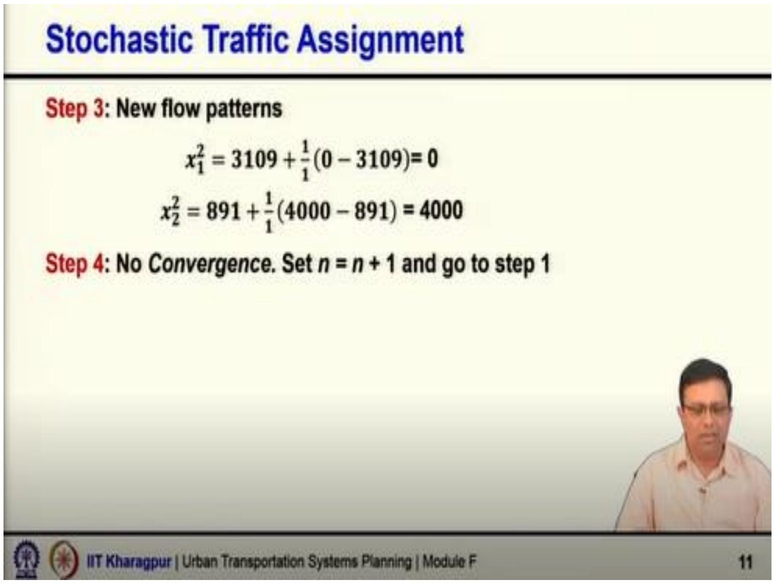
And then when you apply MSA and show the results for intermediate thing we are assuming that theta value is 1 minute to the power minus 1.

(Refer Slide Time: 21:52)



So, initialize what we know the zero flow travel time 1.25 and 2.5, so accordingly you do the stochastic loading and you find that x 1 is 3109, x 2 is 891. Then with 3109 as x 1 and 891 as x 2, we calculate revised travel time t 1 and t 2 for iteration 1 and using this given link cost function and you find out t 1 is 286 and t 2 is 3126. So, the stochastic loading again is done obviously the auxiliary flow will go to all t 2, that is what is the done. So, y 1 is 0 in this iteration and y 2 is 4000 because t 1 and t 2 is like this.

(Refer Slide Time: 22:57)

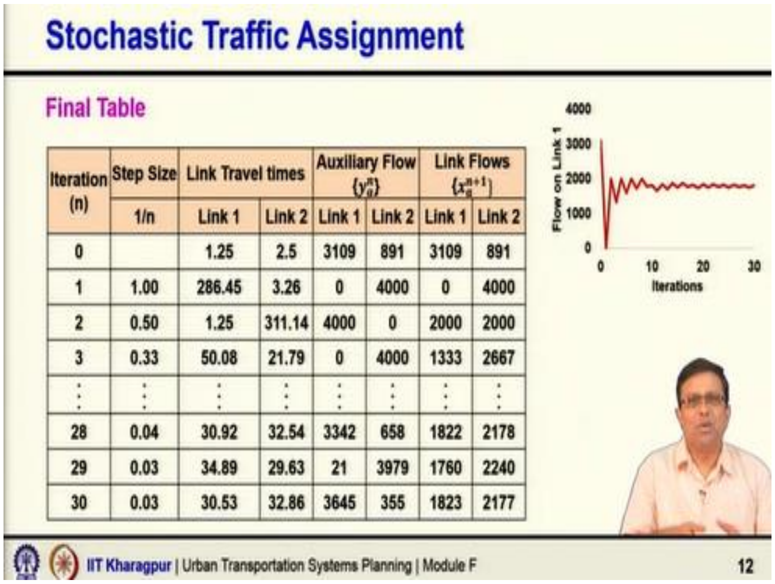


Then, new flow pattern you calculate exactly using the same formula as I said x a n + 1, for n + 1th iteration. What will be the actual flow we are updating? That is equal to what was the flow in the nth iteration plus a change, what is that change? That change is 1 by n within bracket, what is y n a auxiliary flow in nth iteration for that link or that path minus what was the actual link flow in nth iteration.

So, here you do exactly the same manner. So, you calculate x 1 as 0, x 2 s 4000 now no convergence. So, changes very significant obviously the first iteration there will be a big change. And then you set n = n + 1, go to step 1 that means again we calculate the travel time and then again do the stochastic loading and this time it will not be 0 and 4000 because of this so huge difference in the travel time.

So, it will not be again zero both will get some flow that is the auxiliary flow and again, will come back and update the flow for the n + 1th iteration. Like that will keep on doing and when we find the maximum change in flow from n and n + 1th iteration is less than the error level acceptable error level or no significant change in the flow from one iteration to another iteration then we stop.

(Refer Slide Time: 24:54)



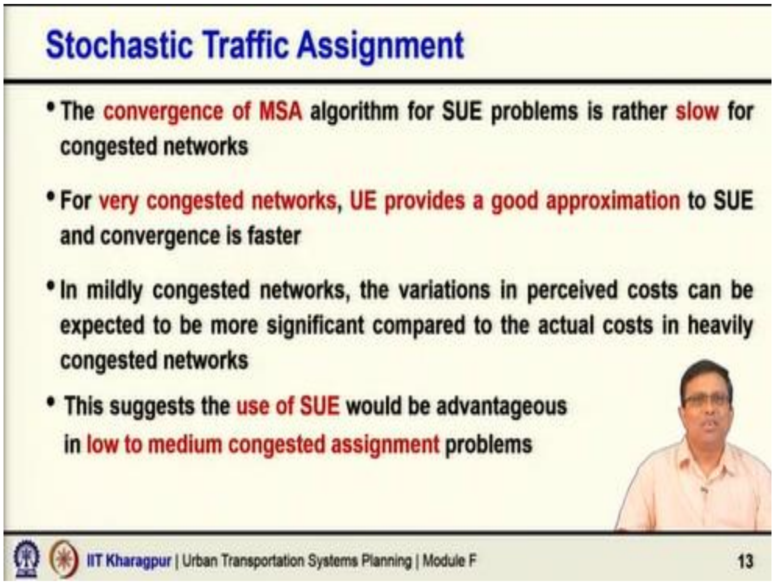
Iteration (n)	Step Size	Link Travel times		Auxiliary Flow $\{y_a^n\}$		Link Flows $\{x_a^{n+1}\}$	
	1/n	Link 1	Link 2	Link 1	Link 2	Link 1	Link 2
0		1.25	2.5	3109	891	3109	891
1	1.00	286.45	3.26	0	4000	0	4000
2	0.50	1.25	311.14	4000	0	2000	2000
3	0.33	50.08	21.79	0	4000	1333	2667
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28	0.04	30.92	32.54	3342	658	1822	2178
29	0.03	34.89	29.63	21	3979	1760	2240
30	0.03	30.53	32.86	3645	355	1823	2177

So, let me now show you the table, a very similar one. So, we start with 0th iteration 1.25 and 2.25. So, you get the auxiliary flow, link flow and then as you proceed the link, I will times are calculated based on the updated flow what you get in the last two columns link 1 and link 2. Accordingly, the next row or next iteration travel times are calculated auxiliary flow are assigned and as you can see initially maybe something is zero but as you proceed finally auxiliary flows you are doing actually stochastic loading.

So, both Links are getting some values and you are doing the update and finally you stop and then if I want to just for example, if I want to show how the link flow has is bad has varied across of different iterations, you can see here x axis shows the number of iteration and y show flow in link 1. So, initially the fluctuations are high and then slowly, slowly, slowly the fluctuation becomes smaller and smaller and the value get stabilized.

So, that means iteration to iteration the change is not so significant. So, you can say that the flow has stabilized or the equilibrium has reached, but even in this case the travel time on alternative paths will not be same as you can see here. So, that is the difference between the change in the flow got minimized and minimum value at reached. So, no significant change in flow iteration to iteration that we have achieved but the travel time will still be different. So, that is what it is.

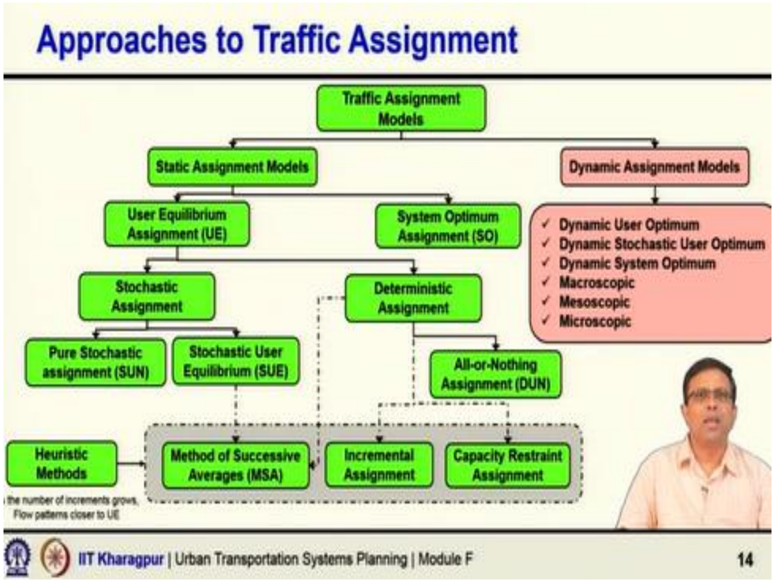
(Refer Slide Time: 26:58)



So, the convergence of MSA algorithm and SUE problem is rather slow for the congested network and for very congested network, it is found that the user optimal equilibrium provides a good approximation to stochastic user of equilibrium and the convergence is faster. So, in mildly congested networks, the variation in the perceived cost can be expected to be more significant as compared to that actual cost in heavily consistent network.

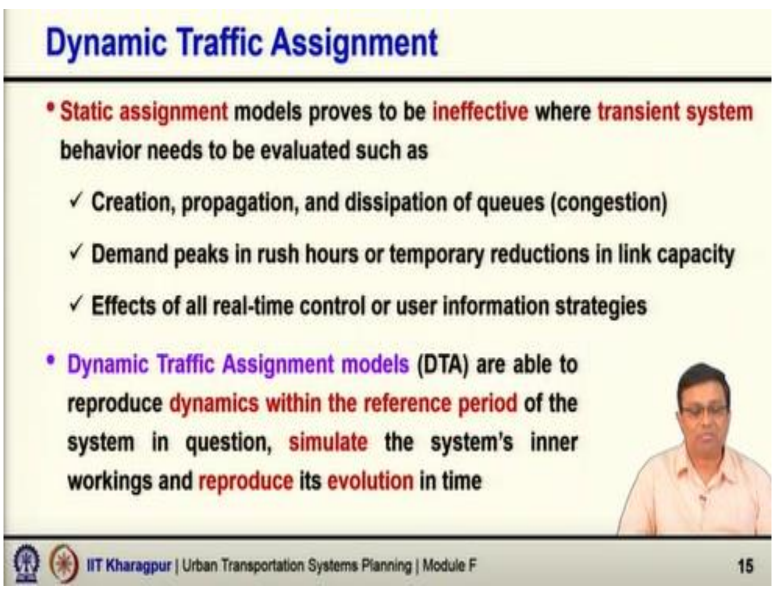
So, this such that generally this is from experience it is found that, this suggests that use of SUE would be advantageous in low to medium congested assignment problem. So, if the network is heavily congested then the difference may not be that significant. So, for very congested network UE provides a good approximation of SUE.

(Refer Slide Time: 28:12)



So, we have almost come to the end. So, this static assignment models all boxes are green now what is remaining is the dynamic assignment model. So, if you few minutes we shall spend now on dynamic assignment model before it close this lecture.

(Refer Slide Time: 28:27)



Now, Static assignment model proves to be ineffective when transient state or transient system behaviour needs to be evaluated. For example, creation, propagation and dissipation of queues in the process of congestion or in the context of congestion building, then demand peaks in rush hours and temporary reduction in link capacity or effect of real time control or user information strategies transient system that means.

What you are trying to mean? When time enters as a variable all what we said for a reasonable period of time when the demand is not changing, network is not changing, nothing is really changing state remains same over that period of time, then what is really happening during that period? But in reality, what happens? Everything is also function, the time is another dimension. You go to a network in the morning 8 o'clock it may be, you know, not much traffic.

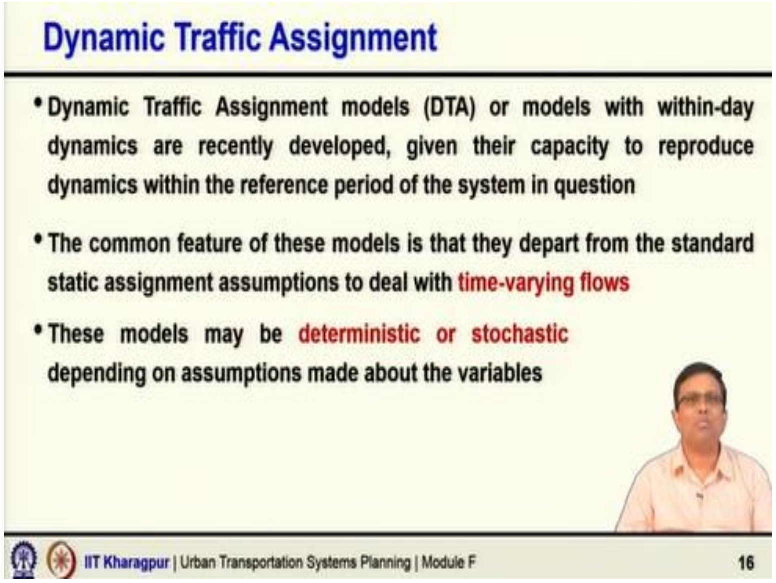
But, then traffic builds up peak hour is approaching the period is approaching then again peak period goes slightly going down. So, the whole process is function of time demand varies over time. So, in that process so one is that it is a function of time and what you do at time t will impact the network state at time t + delta t. So, how the traffic is been distributed at time t will have an impact on the state of this alternative paths at time t + delta t.

So, t + delta t decision will also depend based on that whatever is the demand at t + delta t time, what happened and t time what was the demand and then how the route choice happened and all that. So, the time enters as another important consideration. Everything is now time dependent, that means whatever assignment we all did we applied it only one time. Now, the one-time assignment will not work.

So, repeatedly we have to do it over a period of time update and then proceed further. So, dynamic traffic assignments are able to reproduce dynamic within the reference period. So, if we say 8 to 12 the dynamics the demand is not going to be same from 8 to 12 in the more starting from 8 am to 12 noon. So, DTA are able to reproduce dynamics within the reference period of the system in question, simulate the systems inner working and reproduce it is evolution in time.

Time is a very, very important thing. So, everything now becomes a function of time. So, we say it we said it, it is maybe if always it is like this, then how things are going to happen? Then deterministic assignment techniques are all fine nothing was wrong but then in reality some more we are trying to be more and more realistic. So, if you consider a more realistic situation if you want to consider then you have to consider the dynamics and you have to accept that it is a function of time.

(Refer Slide Time: 32:36)



So, Dynamic Traffic Assignment models or models within day dynamics are recently developed, that is where we stop here. So, the all still now it is evolving, given their capacity, why they are so important these days or in the modern era of traffic assignments or traffic control? Because given their capacity to reproduce dynamics within the reference period of the system in question. The common feature of these models is that they depart from the standard static assignment.

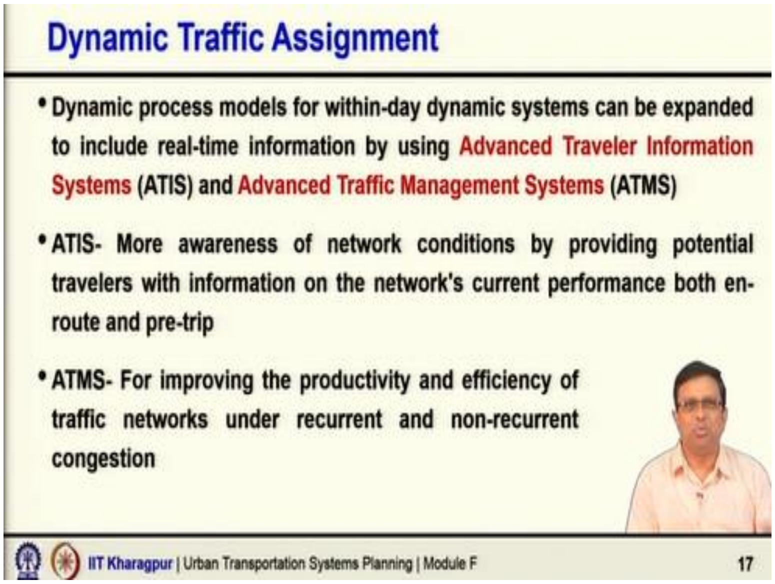
As I said, earlier those assumptions so deal with time variant or time varying flows. As I said, it is building up 8 o'clock something 8:15 something 8:30 something the demand is also not same. Demand is also changing over time. So, how the decisions are the way the decisions have been made at time t what is the state that will also impact the traffic state at t + delta t and therefore, also will influence the decision making at t + delta t.

So, these models may be deterministic or stochastic that again. Now, you do consider, this is the third dimension. So, if you, so this is the time function. So, three aspects initially we say perfect knowledge no perception error, deterministic, no capacity constraint part and no variation over time demand variation over time then, we say deterministic and stochastic one considers the change in the perception the error in the perception.

Because nobody has got the perfect knowledge it is all percept time, percept cost, percept route characteristics. So, the stochasticity comes into picture capacity constraint then coming to picture and we ended earlier when we discussed about our stochastic dynamic a stochastic user equilibrium that we are considering stochasticity and considering also the capacity constraint. Now, this is a new dimension function of time, demand itself is function of time.

So, everything whatever you have that we cannot do one time but things are happening again and again every time every delta t time the decision making are happening. Now, once you call dynamic you are actually focusing then on the time dynamics time variant flow then assignment you have options you can consider it deterministic assignment, you can consider inside against static a stochastic assignment, whatever it will be your assumption, you can do both.

(Refer Slide Time: 35:57)



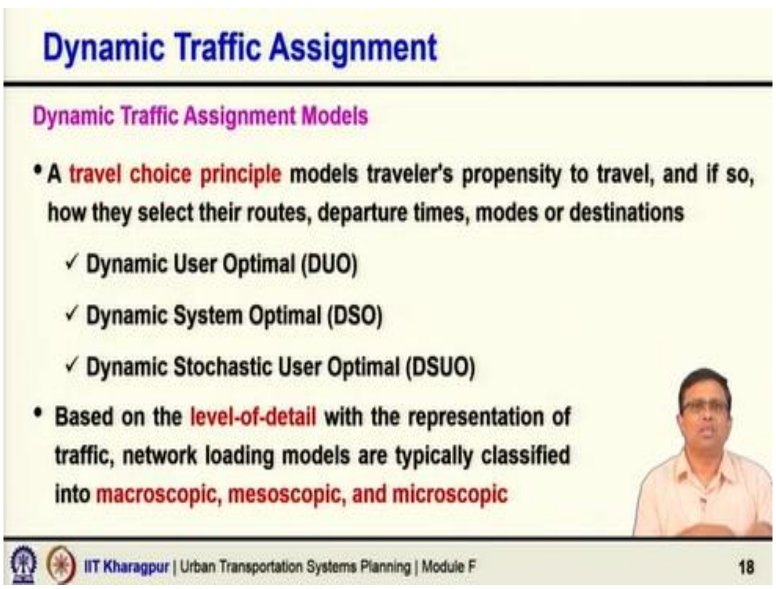
Now, dynamic process models for within day dynamic system can be expanded to include also we can expand to include real time information by using Advanced Traveler Information System what is popularly known as ATIS and also Advanced Traffic Management System ATMS. Real-time if you provide information pre-trip information in route information what type of every type of information will have different impact.

What it normally used to be? Based on historical experience that is one type of information. Predictive it is another type of information; I am only giving the instantaneous traffic information that is again one type. So, which way you display information? Qualitatively, quantitatively so, many things everything will have an impact. So, as I say ATIS is more awareness of the network congestions by providing potential traveler with information of the networks current performance both en- route and pre route.

And similarly, ATMS for providing the productivity and efficiency of the traffic networks on the recurrent and non-recurrent congestion both. What is recurrent congestion? Every day, the traffic is volume is high to congestion occur that is recurrent every day is happening. What is non-recurrent? Suddenly one vehicle breakdown has come. So, suddenly this congestion happens, it does not happen, not that that location every day, there is a vehicle breakdown.

But recurrent congestion happens every day or may be a traffic accident, two vehicles met with an accident and therefore the congestion non-recurrent does not happen every day at that location at that time, both can be considered.

(Refer Slide Time: 38:01)

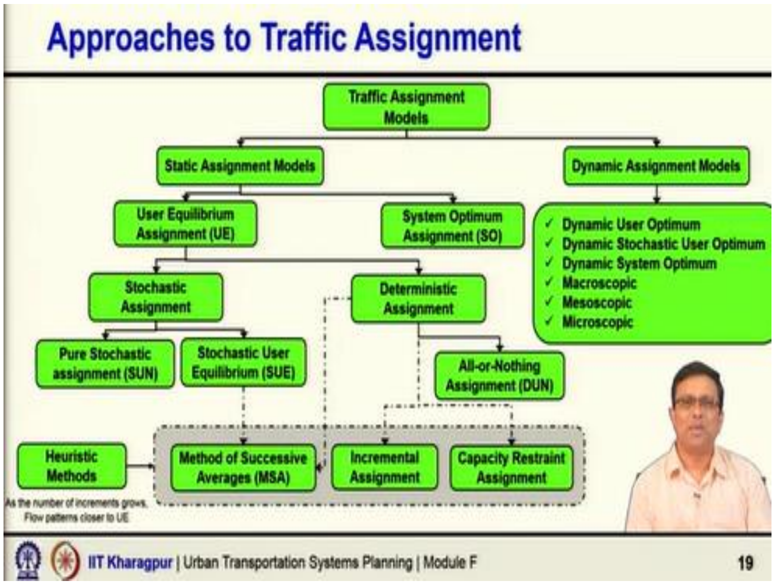


So, depending on what you are doing how you are assigning you can then have it is not user optimal alone but we can say dynamic user optimal because we are doing user optimal but also considering the time dynamics time variant flow or you can be dynamic system equilibrium or also you can have dynamic stochastic user optimal equilibrium, what is say that SUO stochastic user optimal or SUE we say solve the same equilibrium or optimal.

So, here in this case one more term will get added. I am also considering the time dynamics. So, it is not only user stochastic user equilibrium or stochastic user optimal but it is dynamics stochastic user optimal. Now, based on the level of detail with the representation of traffic network loading models are typically classified as macroscopic, mesoscopic and microscopic you can understand by the terminology, we cannot go into further details because we have, the limitations of time and number of lectures that we can schedule.

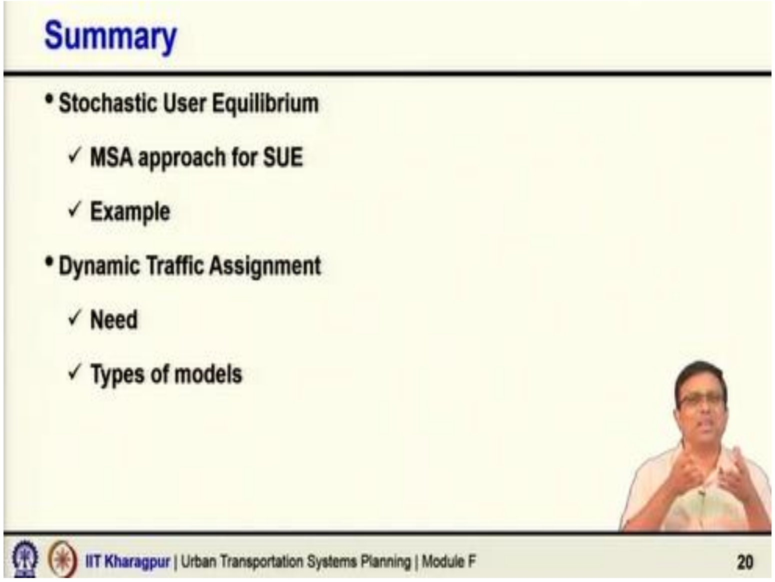
So, macroscopic is at macro level, microscopic is you know each vehicle to vehicle level more most little level and mesoscopic is somewhere in between all different models are available and different platforms are also available.

(Refer Slide Time: 39:33)



So, with this I would say that, now you look at this flowchart, all are green all boxes. So, we try to give you an overall understanding of various approaches to traffic assignment. Some we could deal what are the fundamentals we could deal with in details and whereas some of the cases I had to leave only with the basic concept like mostly the dynamic traffic assignment model. But as I said the still evolving and, lot of you know things are there for you to know. So, if you are interested do some self-study and look at some of the research paper to learn more.

(Refer Slide Time: 40:19)



So, with this I conclude so what we discussed here is the stochastic user equilibrium the formulation and how to solve using the using stochastic equilibrium and then MSA to get the stochastic user equilibrium. Now how this equilibrium is different from deterministic user equilibrium, and also gives you a little bit introductions to time dynamics. We said why we need it, and what is the basic thing new? Is the introduction of this time dimension in the whole work?