Urban Transportation Systems Planning
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Lecture - 49
Stochastic Traffic Assignment I

Welcome to model F lecture 9, in this lecture we shall discuss about stochastic traffic assignment techniques.

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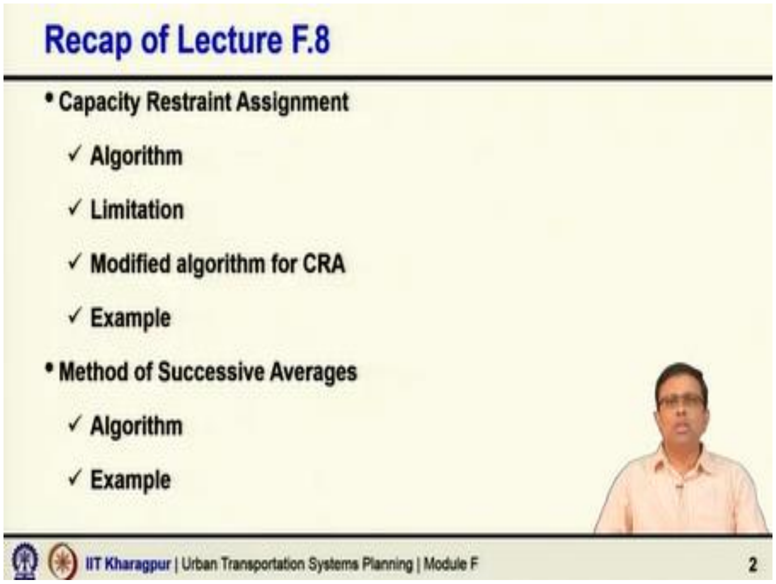

In lecture 8, we discussed about capacity restraint algorithm, the regular capacity restraint algorithm and also the problem or the potential problem of the flow flip flop and then the modified algorithm for capacity restrained assignment to handle that kind of problem. Took an example to tell you step by step how this algorithm can be applied and to get the solution, then we also discussed about the method of successive average, explain the algorithm the steps.

And took an example to demonstrate that how that method of successive average can be applied to solve or to get the solution for a given situation.

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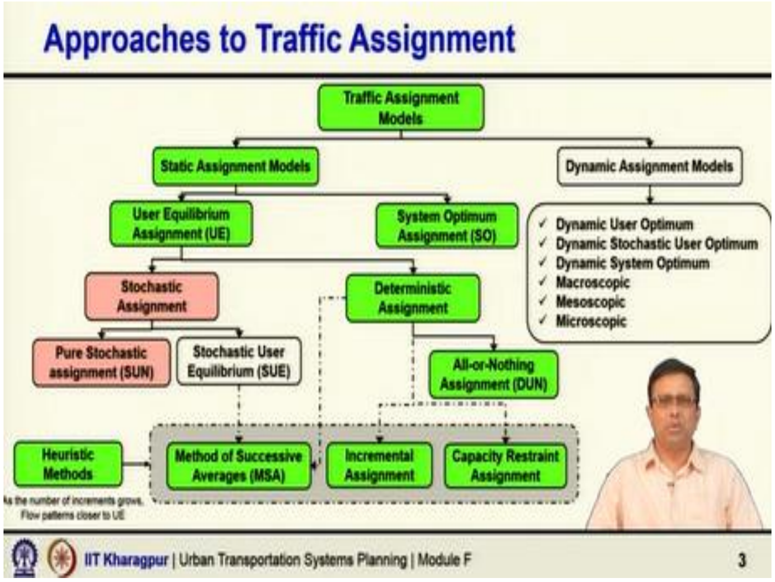

Now, in continuation to our lecture or the discussion on this traffic assignment topic we shall now go to stochastic assignment. This is already you know, the ninth lecture so we are happy to see that most of the boxes are marked as green we have already covered. So, now we take up this stochastic assignment in this lecture and then specific under stochastic assignment the pure stochastic assignment that algorithm or that approach we shall describe.

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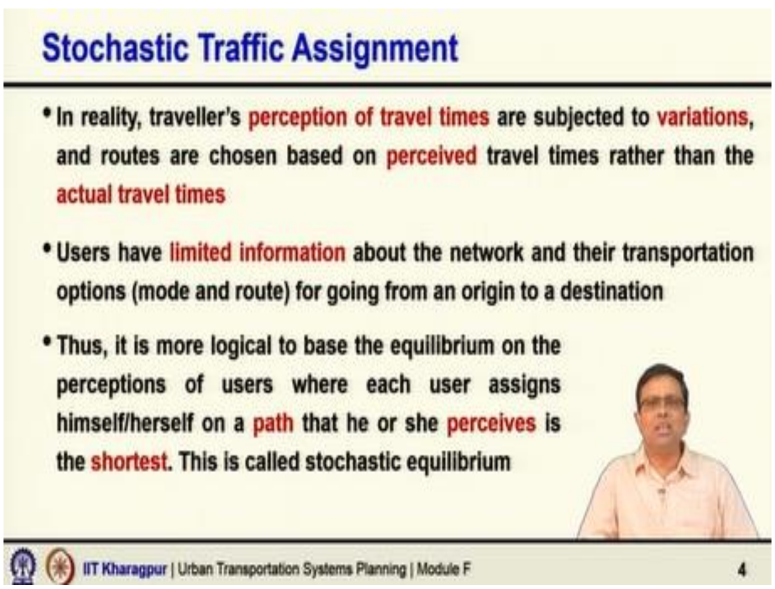

So, far we said that in user optimal equilibrium nobody or no user would be able to choose a route where the travel time is lower than the present travel time for the chosen route and that if nobody can get a better route no individual can get a better route then we said that the network is in user equilibrium because every user has got the best deal. Now, if you rethink what is this travel time which is a 10 minute 15 minute.

But then how people take decision, anybody tells all user uniformly that exactly the current travel time is 10 minute or current travel time is 15 minute, answer is no. Then how the route is taken or route choices done by individual? It is based on the perceived travel time and you will accept that the perceived travel time, when you choose a route you choose based on your understanding of travel time.

What is the travel time along route A and what is the; long travel time along route B as you perceive and then you choose the one which you think will minimize your travel time. So, in reality travellers perception of travel time are subjected to variations, because a traveller who travels every day and a traveller who travels may be occasionally, a traveler who travels you know once in a while, everybody make decision everybody every individual makes a decision.

But their decision is based on their own perceived travel time, whatever they got information you traveller may be I will check some website and then public domain whatever information is available or if I know my friend I may ask and then once I travel based on my own experience I will decide how much travel time it takes. So, these travel times perceived travel times are subjected to variations.

Therefore route chosen are based on in reality based on perceived travel time, then the actual travel time. Who knows the actual travel time? So, it is on the perception. Now users have limited information about the network and their transportation option what modes are available of course in the context present context, we are talking about the route for going from an origin to a distribution.

So, therefore whatever we said so far travel time 10 minute, travel time 15 minute if we consider that they are the true travel time. Now, the question is we rethink and we question ourselves that you know, what is this travel time? What does it mean in reality? If somebody is making a decision and we realize that it is actually the decision is taken based on perceived travel time and the perceived travel time will not be same to every individual due to so many reasons.

Limitations in information, their own experience their own knowledge about the network about the alternative routes and so on. So, and also sometimes the decision may not be based on only travel time but may be based on you know, some other factors as well as you have seen the description model when we talked about in the context of mode choice the probabilistic choice model, we talked about the first about the deterministic model and then probabilistic model.

So, you remember, recollect and remember all the points what we mentioned there, the possible error. So, here also there will be possible errors and the perceived travel time may be very different from the actual or the true travel time. Because of the true travel time, users may not even know, so all together then there is a need to relook at this user equilibrium. In the sense that we should we know now, whatever routes happens is not does not happen based on the actual travel time.

But, it is based on the perceived travel time by every individual say every individual makes a decision tries to optimize or minimize him or her own travel time but her own perceived travel time not the actual travel time. So, it is more logical to base the equilibrium whatever we said as user equilibrium on the perception of users where each user assigns himself or herself on a path that she or she perceives is the shortest, this is basic consideration in the stochastic equilibrium.

So, with that assumption whatever user equilibrium will reach now, it is still user equilibrium. But every individual is trying to minimize his or her own perceived travel time, not the actual travel time.

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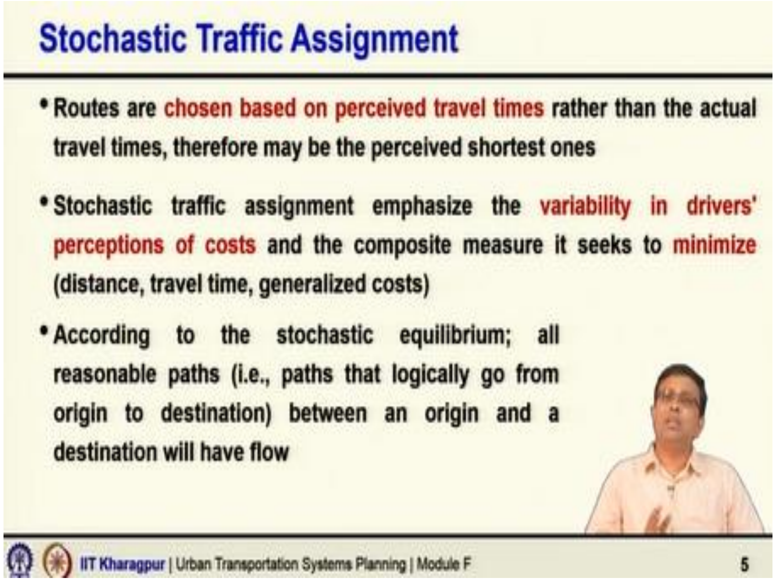

Routes are chosen in reality and as per this assumption or realization based on perceived travel time rather than the actual travel time and therefore may be perceived shortest cost, the shortest route rather than actual shortest route or shortest path. So, everything is perceived that travel time is perceived travel time and my chosen or the shortest path is my perceived shortest path. So, stochastic traffic assignment emphasizes the variability in drivers perception of cost.

Cost is a general representation of deterrence it may mean travel time it may be distant or the cost travel cost whatever you say and also the composite measure it seeks to minimize as I say, sometimes it may not be the travel time alone, but travel time and distance and maybe also the generalized cost and so on. So, according to stochastic equilibrium all reasonable parts that mean paths that logically go from one origin to one destination.

In a practical manner between an origin and the destination will have flow because of this variability in the travel time percept travel time and because that the perception varies widely. So, everybody is trying to select route based on perceived travel time. So, obviously it will be away from reality and the real shortest path only will get loaded that is not going to happen, multiple paths will be used because of the variation in the percept travel time.

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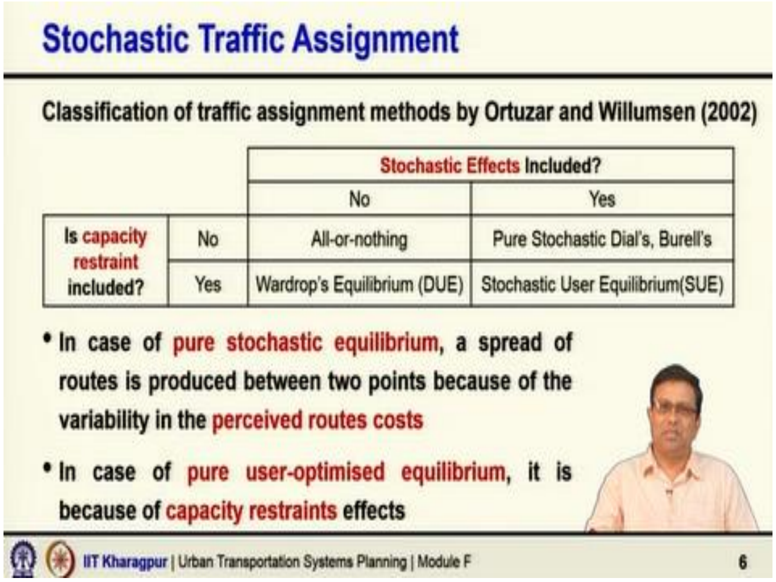

So, we can also look at this classification, which is quite interesting is capacity restraints included that is one question the answer may be yes or no is stochastic effect included the answer may be again yes or no. Now, if we are not considering capacity and if we are not considering the stochastic effect then it is all or nothing, everybody takes the shortest path, perfect information, no variation in the perceptive travel time, everybody knows the exactly the true travel time, there is no capacity restraint.

So, there is no variability in the perceptive travel time and there is no capacity constraint, so everybody goes along with one path that is shortest path and no flow takes place in any other path. Now, we do not consider stochasticity, but we consider the capacity restraint, assume that the routes or paths have got they are all physical infrastructure so they have limited capacity and depending on the flow.

The flow may be even more than the capacity or even if it is not more than the capacity then also because of conjunction effect the shortest path definition may change and the flow may have to be on a different route other than this only the shortest path and through that so that means I am considering the capacity constraint, but I am considering that there is no stochasticity, no variation in the perceptive time.

Then I get deterministic user equilibrium and that is what we initially told you as Wardrop's equilibrium. So, we call it DUE, deterministic user equilibrium. Now, then the other two are remaining, I do not consider capacity constraint, but consider the stochasticity of the variability in the perceptive travel time. Then it is pure stochastic algorithm, Dial's algorithm or Burrell's algorithm, we have not yet discussed but you will know them gradually some of those.

And then if you consider the stochasticity effect that the variability is likely to happen that what is the reality and also the capacity constraints will be there that is also reality. Then whatever equilibrium we are looking for or we are expected to get in reality that is stochastic user equilibrium, it is not deterministic user equilibrium, but because the stochasticity is considered so it is stochastic user equilibrium.

Now, in case of pure stochastic equilibrium, that means we are not considering the capacity constraint, but we are only considering the variability in the perceived travel time, is spread of the route is produced between two points because of the variability in the perceived route cost. Again, I said the cost is a general depiction of deterrence it may be you can call it perceived route travel time or perceived route travel cost whatever it is.

Now, remember that in case of pure user optimized equilibrium or user optimal equilibrium, it is because of the capacity constraint effect. Why the route is not only the shortest route when I do not consider the stochasticity, you assume that there is a perfect knowledge. Everybody knows the correct travel time along every alternative path. So, there is no question of any variation in the perceived travel time.

In that case, also multiple paths are taken why multiple paths are taken in DUE it is because of the capacity restraint effect, capacity consideration. Driven may be more than the capacity or driven may be even less than the capacity but still the condition effect will be there. So, multiple paths will be chosen end of the day multiple paths will be chosen but in stochastic user equilibrium, stochastic equilibrium not stochastic user equilibrium.

Pure stochastic equilibrium still multiple paths will be chosen, but the reason is very different that fundamental should be very clear that reason is variability in the perceived travel time or perceived travel cost. Nobody, you know people do not have idea about the true travel time or true travel cost it is just the perception, perceived travel time. So, perceived travel time likely to vary.

So, multiple paths will be chosen by people because they will think those corresponding paths are the shortest path that shows their perception. So, both cases we are likely to get multiple paths which will get which will have flow more than 0, but pure stochastic it is only due to the variation in the perceived travel time that is what we say due to stochasticity and in pure user optimal equilibrium rather DUE, multiple paths will be there again the multiple paths will be chosen.

But, the reason is very different reason is the capacity restraint effect or the capacity consideration and conjunction of the limited capacity consideration.

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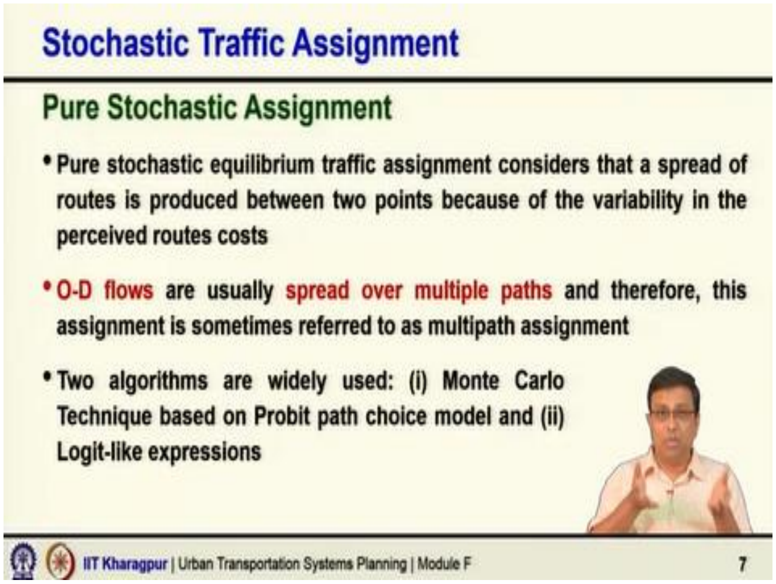

So, with that let us go to the pure stochastic what is happening? In pure stochastic equilibrium traffic assignment, we considered the spread over the route and it is produced the spread is produced between two points of the variability in the perceived route cost. That means multiple routes will be chosen because of the variability in the perceived root travel time or root travel cost.

O D flows are usually spread over multiple path because as I said, there will be variability in the travel time, so not everybody will not have the exact same information. So, everybody's perceived shortest path will also be different, so some people will think path A will give the optimal or the shortest path or the travel time minimum travel time, somebody will say path B somebody may think path C, this variability is expected that is what is the reality.

So, O-D flows are usually spread over multiple paths and therefore this assignment sometimes is referred to as multipath assignment because multiple paths flows are there. Even when there is you know, the demand is insignificant as compared to capacity, I bring another consideration because if you see this you consider a network where the demand is not so high as compared to capacity.

Still there will be multiple paths where the flow will be distributed just because of the perception, perception in the variability and the perceived travel time. So, two algorithms are approaching widely used one is the Monte Carlo based approach on Probit path choice path and the other is Logit like expression. You are very familiar with the Logit like expression, Probit model we have not covered but we said that what assumption of error.

What distributional assumption lead to Probit model at least we said that and we say the other estimation complexity associated with Probit model. So, Monte Carlo is technique also can be used for Logit like expression.

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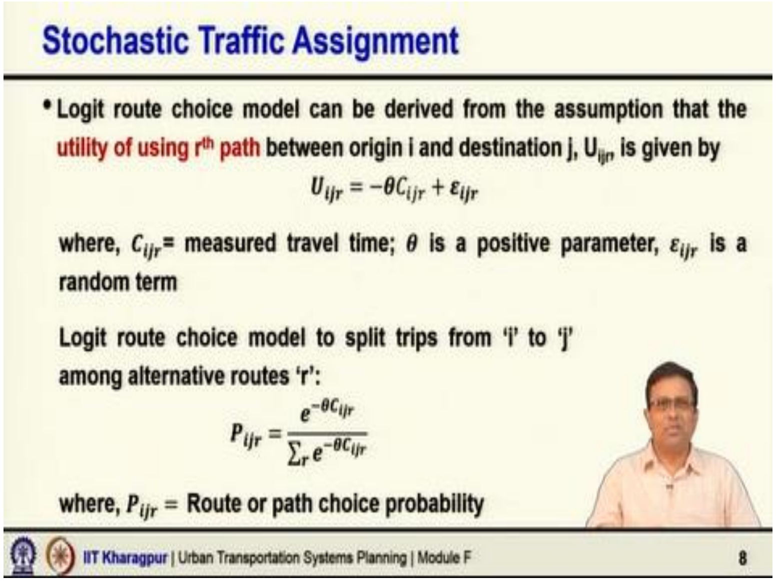

Here, we will talk about the Logit like expression, this is known to you so now the remaining thing becomes little simple. We are considering that utility of using r path, utility of travel between i and j along path r, so that is what it is written as U ijr equal to minus theta C ij, C ij is the cost of C ijr, so cost of travel it may be travel time travel costs analyzed cost, whatever you take.

So, theta is a coefficient and for a parameter positive parameter and C ijr is the cost of travel between i and j via or through route plus there is an error term. So, the true utility is measurable portion of the utility that is measurable portion is the C ijr plus an error term. So, how the Logit routes will happen what is the probability of taking a particular route, r while traveling from i to j it is P ijr equal to e to the power minus theta C ijr divided by e to the power minus theta C ijr, sum over all r sum over all paths r there are multiple paths so sum over all r.

This is very much like the Logit model what you already know and we have discussed in the context of Mode choice and the probabilistic charge model is very similar expression.

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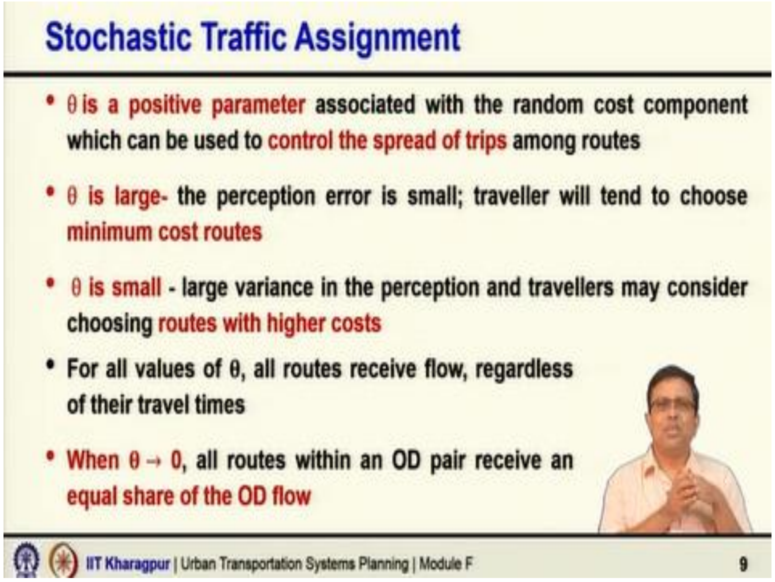

Now what is theta? The theta is a positive parameter associated with the random cost component which can be used to control the spread of trips among routes. I would generally say that it is a positive parameter associated with the cost component and this theta value can be used to control the spread. For example, if the theta is large, a large value of theta with that when you actually do the assignment what it means the perception error is small.

And therefore traveller will tend to choose more the actual minimum cost more likely to choose actual minimum cost or we can say still the variable will be there, but the variability will be less as compared to the theta value which is small. So, theta value if it is small higher the smaller the theta value the larger will be the variance of the variability in the perception and travelers may considering routes.

You know more likely to consider routes with higher cost or more and more traveler will go there. Now for all value of theta, theta can be have any value any positive value is said, all routes receive flow regardless of their travel time. Only thing the fraction or the shear how much of the total demand what fraction will come to a route or what fraction will get distributed even to higher cost route, that will get control by what is the value of theta.

So, as I say if it is large then more and more will take actually the shortest path and the variability variation or routes with higher cost will get probably lesser share but if the theta is small more and more shear will go to the higher cost route and when theta tends to 0, almost like 0 that will mean more and more I said that it will go larger variation. So, it will mean that all roots in the O-D pair almost will receive an equal share that we will end up with theta equal to 0.

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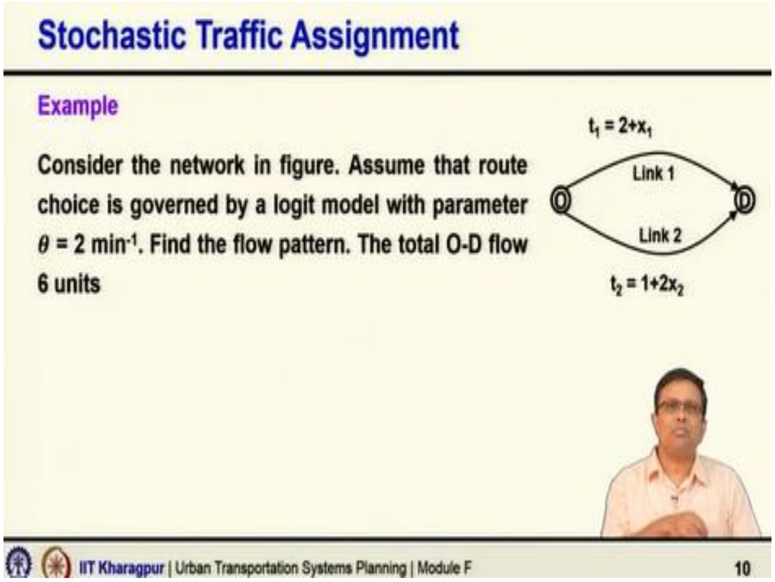

So, consider an example quickly that we have an O-D flow. So, link 1 link 2, two travel time functions are given and let us say theta equal to 2 minute to the power minus 1. So, value of theta is actual value is 2.

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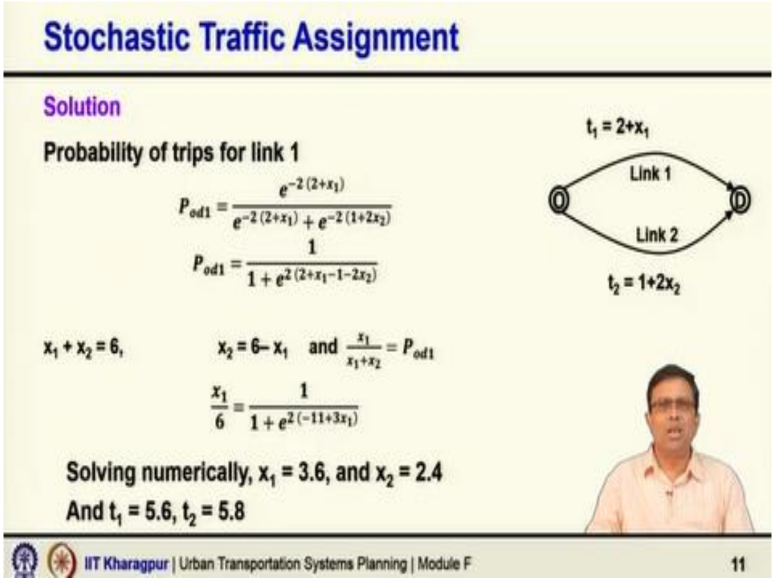

Now, you want to distribute O-D flow. So, you simply calculate, you know that logic like expression what we said earlier. I will go back to that slide here, probability of P ijr how much here it will get while traveling from i to j. In this case only two nodes are there 1 to i, 2 to j or 1 to 2 and two parts are equal to 1 are equal to 2. So, you know e to the power minus theta ijr divided by e to the power minus theta ijr sum over all routes.

So, in this case, sum over route 1 and route 2, you get that. So, that is what you get but you also know x 1 plus x 2 equal to 6 the total 6 unit flows are to be distributed. So, x 2 equal to 6 minus x 1, so you do that substitution and also, you know probability of taking path 1 will be what x 1 by 6. So, x 1 by 6 equal to 1 plus e to the power 2 within bracket minus 11 plus 3 x 1 that comes out the substitution.

So, it is a simple calculation not a big thing and once you solve you find x 1 equal to 3.6 and x 2 equal to 2.4 and the corresponding travel times are 5.6 and 5.8, not exactly same. We do not expect it, because of variability or because of the error in the perceived travel time. So, that is what the solution what we get.

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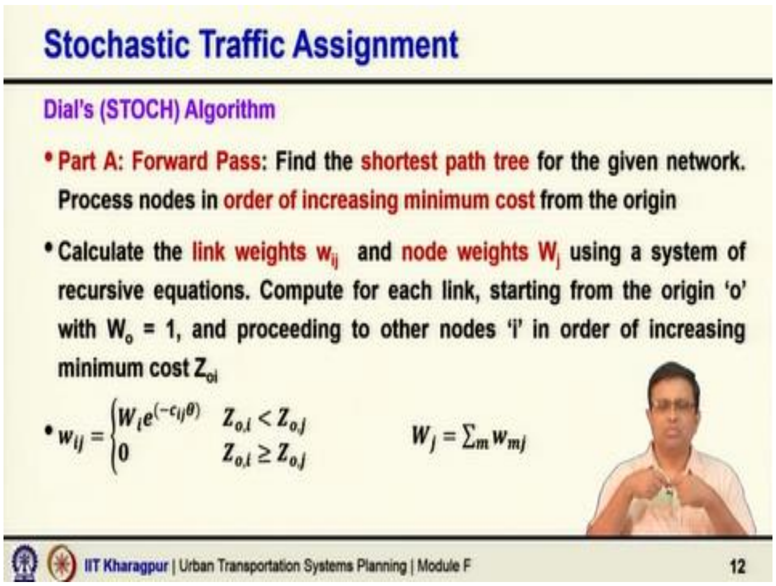

Now, we go to Dial's algorithm to tell you how you applied, it works in two parts; part A and part B. Part A understand it, it is not complicated you can and I shall also take an example to show you step by step how the calculation are done where that will be more convenient and comfortable for you to understand it very clearly. But let us state that algorithm and then we go to an example to show you the application.

What is the forward pass? What we do? We try to find out the shortest path 3 for a given network and we start from one node. So, one node to all other node we find out the shortest path then process nodes in order of increasing minimum cost from the origin. So, if my origin is one and if I am taking one to two, one to three, one to four, one to five like that, I find out the shortest path cost and then I shall process the destination nodes as per the increasing minimum cost.

So, that means among all the nodes whatever will be the starting node always will have the list and then all others. So, others also the cheapest or the minimum cost to a destination that destination will be processed first followed by the immediate next higher cost whatever may be the destination that destination result process and starting from the origin O with W O equal to one, where is the origin starting that weightage will take one and processing of other nodes i in order of increasing minimum cost.

So, how what will be the value of weightage will calculate W ij,W ij will be W i into Wij equal to W i that means where is my i node, e to the power minus theta C ij or zero depending on whether j, y is less than j o j. Who is the origin node? And, we are talking about link i j. So, if jth y less than jth o j then W i j will be W i into e to the power minus theta C ij and if the jth y j is greater than jth o j then it will be zero.

And then finally what will be the total weight assigned to node j, it is somewhere all the lengths which are coming and their weight. So, it is W mj assuming from various same the connection is our connections are coming to j. So, what will be the sum of all the weights of the links which are connecting to j.

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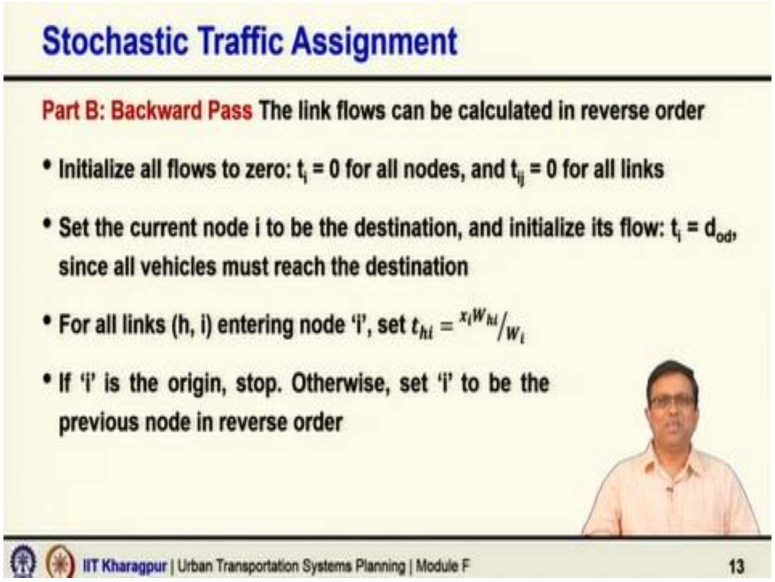

Then B is the backward pass, in backward pass what we do, the link flows can be calculated in the reverse order. We start with the destination link for this one where the finally the flow will reach and initialize initially all flows to zero and t i equal to zero, t ij equal to zero that is the initialization and then set the current node i to be the destination and initialize it is flow and then come back slowly.

I think the remaining part, I shall explain with the example and then later on you can come back and see this steps you will follow it very easily.

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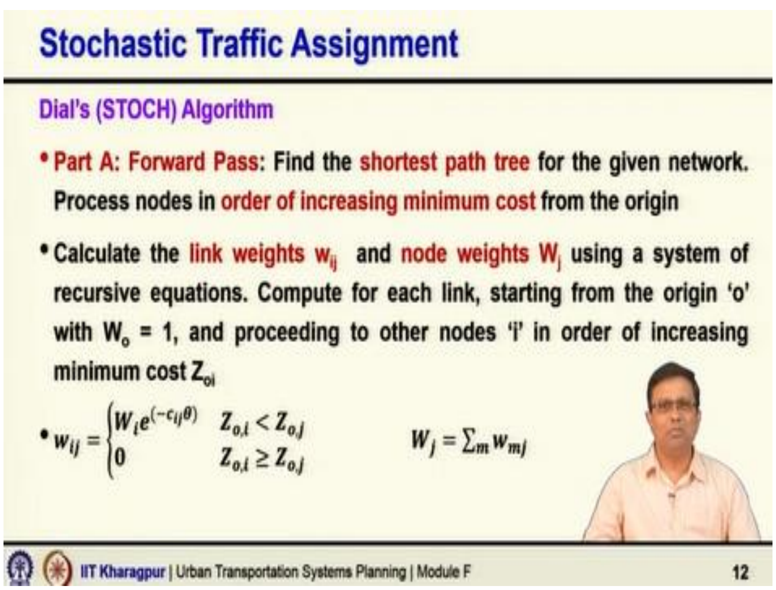

So, let me take an example consider the network in figure with travel times given on the links. Assume that the route choice is governed by a Logit model with parameter theta equal to 0.5 minute to the power 1, minus 1 and find the flow pattern and the total flow from node 1 to node 4 is 1000 units. So, we want to assign 1000 unit flows from node 1 to node 4.The first what we will go step 1 and step 2.

Step 1 will be forward pass, that is what we said here part A or part B. So, part A is the forward pass, so what we do?

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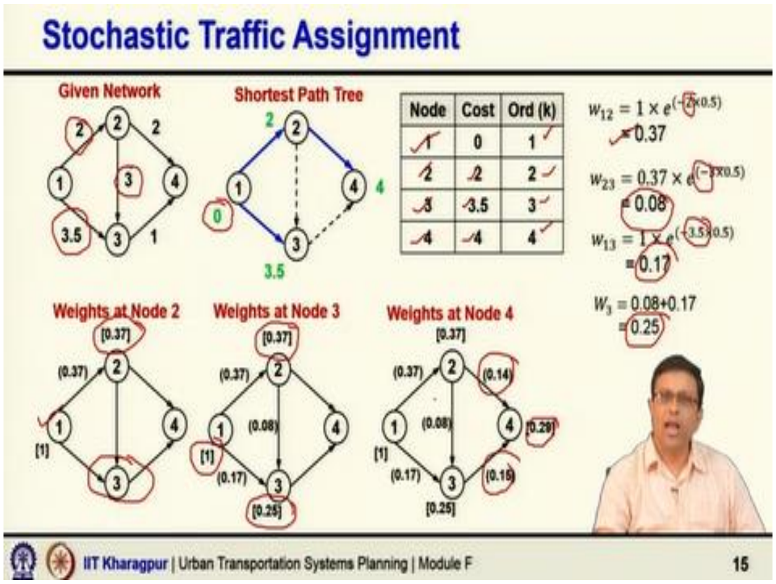

Let us try to understand clearly, we start with node1 and then 1 is connected to 2, 1 is connected to 3, 2 is connected to 3, 2 is connected to 4, 3 is connected to 4 and these are the cost. So, here easily since it is a small network you do not need a separate algorithm to calculate it, of course is the classroom examples. So, I have taken a small example. So, this is the shortest path is known. So, what we are doing? We are arranging them in order.

Remember, the order what we said here, in order of increasing minimum cost, that is what is told. So, what we are doing here. So, you take node 1 starting node, the cost is 0 to order is 1. You see two the shortest path cost is 2, 3 shortest path cost is 3.5, 4 2 plus 2, 4, that is the shortest path. So, order becomes first 1 then 2 then 3, then 4 and what are the cost? Costs are 0 for 1, 2 for 2, 3.5 for 3 and 4 for 4.

So, because of this as per the cost we are putting them. So, the water is 1, 2, 3, 4 then what will we do? We will process these nodes as per their order. So, what will be there, we start from 1, what will be the weight for 1 to I go back here. It is W i e to the power minus theta C ij. So, weight for 1 is 1, 1W i into e to the power minus theta is 0.5. And what is the C ij value, this Cij value is 2. So, e to the power minus 2 into 0.5.

So, you get then 0.37, W 23 what will be for this links next thing W 23 this 2 3 will be 2 weightage for 2 is what then the total which is for 2 will be only this node is coming the other two are going from 2 to 4 and 2 to 3. So, how much we need only this node is coming the other two are going from 2 to 4 to 2 3, so how much total is coming 0.37. So, it is 0.37 into e to the power minus theta C ij theta is 0.5, C ij, j is 3, node three.

Node three what? A node three is three and then 1 3 is 3.1 again, 1 to 3 is 1 and e to the power minus theta C ij. So, C ij is what? Here 3.5 so, here the cost is 3.5.So, here it is 3.5 and when you consider 2 3 this cost is 3. So, you consider this also as 3 and 1 to 2 this is 2 so you consider this as 2. So, like that you calculate W 12, W 23, W 13. Now, once I have calculated let us say what will be then the weight for this node 3, go back.

It will be W mj sum over all m whichever nodes are coming where from there coming. So, 1 to j 2 to j 3 to j like that sum over all those weights now you can see 3 it is coming only from 1 to 3 and 2 to 3 that is all, 3 to 4 is going out 3 to 4 not coming to 3, coming to 3 is only 1 to 3 and 2 to 3. So, 1 to 3, what is the weight 0.08 that is what we have calculated and 2 to 3 and 1 to 3 is 0.17. So, what is the weight for this node 3, weight for node 3 will be 0.08+ 0.17, so 0.25.

So, weight for node 1 is 1, weight for node 2 is just only 1 to 2 is going no other link so 0.37, 3 as we calculated 0.25, so we calculated this weights for node each node 1 2 similarly 3 to 4, 2 to 4 and then this weights are calculated as 2 to 4 as 0.14 here it is 0.15, and then node 4 the total varies 0.14 + 0.15 because 2 to 4 is coming from 2 to 4, 3 to 4 coming from 3 to 4 no other link is coming. So, 4 total weight 0.29.

So, all these are done for exactly whatever is shown here, W ij how we calculate and then how we calculate W j exactly this and this part these are the two components we are considering. So, accordingly we have calculated. So, now we know each link in the network, what are the weights and each node in the network what are the weights? So, for all nodes and all links we have the weights and we process them in which order as per this order.

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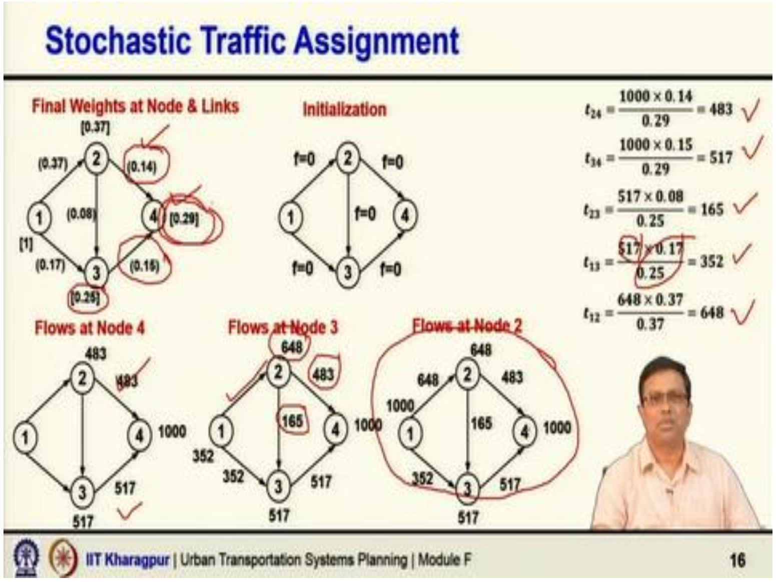

Now, the backward pass, so the forward password will do. So, the backward pass very easy, simple what we do, we know that 1 to 2 how many are coming finally 1 to 4; 1000 trips. So, finally if 1000 trips are coming to 4 weight total weight for node 4 is 0.29 and which are the links that are connecting 2 4 and 3 4, these are the two link 2 4 weight 0.14, 3 4 weight 0.15 and total node weight is 0.29, which is the sum of this thing.

So, what will be then t 2 4 how much will be assigned to 2 4, 2 4 will be 1000 into 0.14 divided by 0.29. So, 1000 into 0.14 divided by 0.29 you get 483. How much will be the 3 4, 3 4 will be this 0.15, 1000 into 0.15 divided by total weight of node four 0.29. So, you get 517. So, you know that this is 517 and you know this is 483. Now once you know, accordingly you do, you know that 517 is coming from 3 to 4.

So, total three flow coming or the demand coming is three where from there coming they are coming 1 to 3 and 2 to 3. So, 1 to 3 is how much 0.17, 2 to 3 is 0.08 and the total weights of node three is 0.25. So, how much will be the 1 3, 1 3 will be 0.17 by 0.25, that is ratio multiplied by 517. So, you see that so what we have done 0.17 by 0.25 multiplied by 517. So, it is 352, what will be the 2 3, 2 3 will be 517 into 0.08 by 0.25.

So, you get 165, what will be the 1 2, 1 2, now you know 2 to 4 is 483, 2 to 3 is 165. So, total how much is coming from two is 648, this 483 + 165. So, 648, so this whole 648 and there is only one link which is coming to 2 so 1 to 2 will have all this 648. So, what is the distribution again the distribution like this. So, thousand is getting originated from one 648 going to two, 352 is going to 3 then from 2 out of 648, 483 are going 2 to 4, 165 is going from 2 to 3 and the 3 is total getting 517 and this 517 is going from 3 to 4 that is what it is. So, the way you get it.

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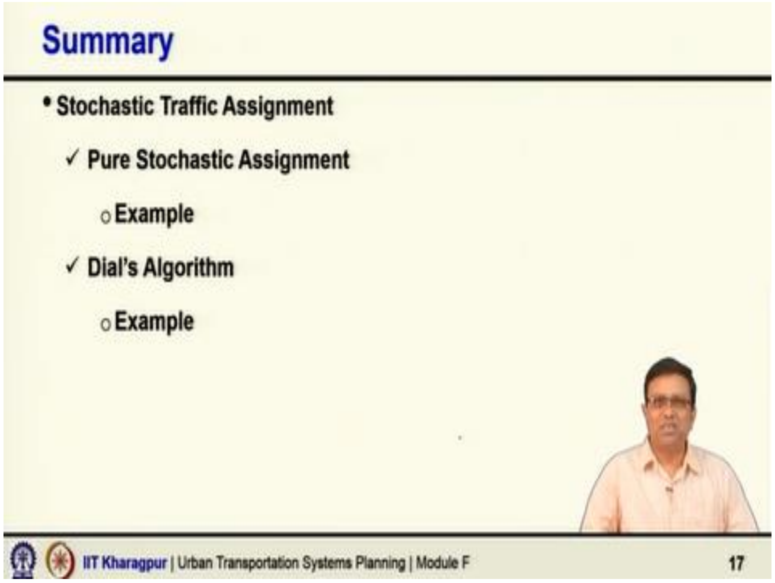

So, what we discussed here is the pure stochastic assignment, why whatever at the stochasticity come from? Why we go first stochastic assignment and then explain the principle and then explain to you the Dials algorithm with an example. So, with this I close this lecture.