Urban Transportation Systems Planning
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Lecture - 47
Deterministic Traffic Assignment - I

Welcome to module F lecture 7. In this lecture we shall discuss about deterministic traffic assignment.

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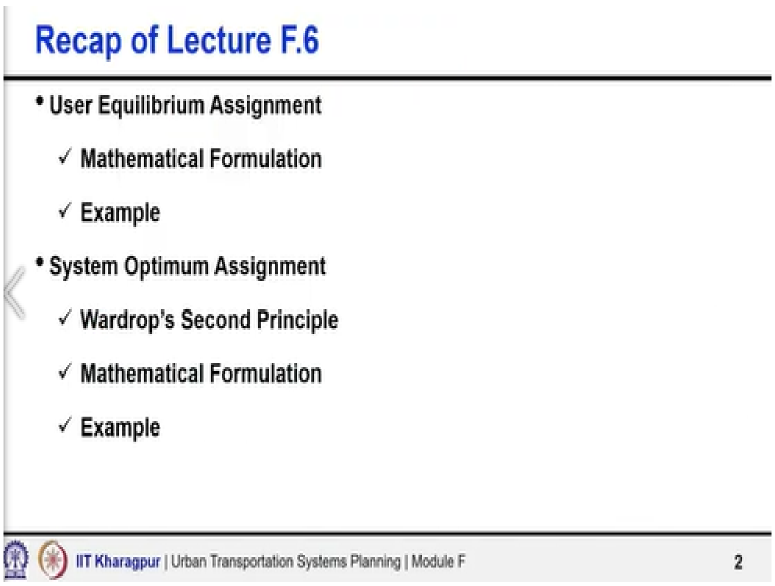

In lecture 6 we introduce to you two equilibrium conditions Wardrop's first principle and second principle which include user equilibrium and system optimum assignment. For both conditions we took one example from user equilibrium and one example to explain you the system optimum assignment. In fact, it was the same example, but we solved the same example assuming user equilibrium.

And also, then as part of the system optimum assignment, and we have shown you how the solutions are going to be different. So, with the concept of user equilibrium and system optimum assignment now we proceed further;

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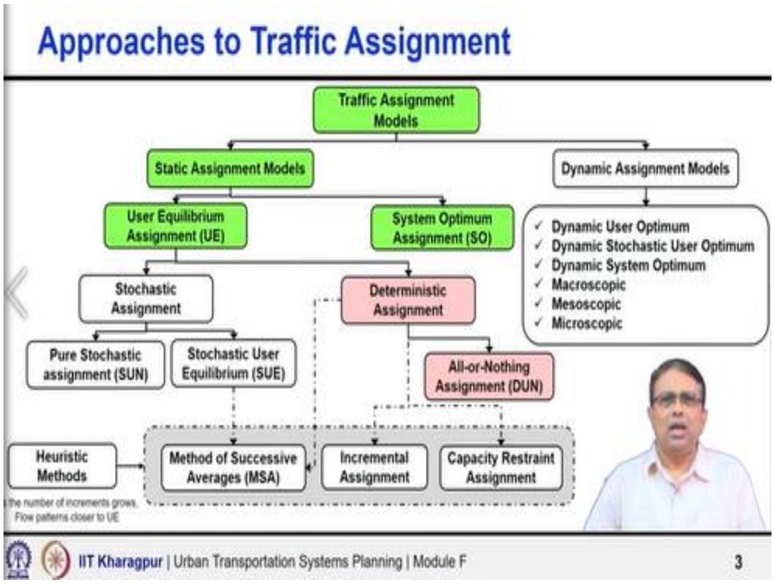

And come back to this mother flowchart once again where we shall now discuss about deterministic traffic assignment and specifically the All-or-Nothing Assignment. Now let me tell you, that you know what is user equilibrium assignment and what is system optimum assignment? The principles are very different. Now whatever we said that in user equilibrium all the routes used by traffic between one origin and one destination.

All such roads which are used by traffic will have equal travel time and any unused route will have either equal or higher travel time. Now this is actually true, exactly in this format what we introduced when we are talking about the deterministic system or deterministic assignment technique. Now the mathematical formulations are there you remember that we try to minimize the area under these curves, demand or the flow and the travel time curves or the cost curve.

And mathematically we have shown you how we can do a formulation. But when there are multiple origins and destinations when there are multiple links then solving it mathematically as we could solve very easily when we have you know just one O-D pair and there are two or three alternative routes that was quite easy. But for a large network where we have you know several origins and several destinations and also, we have so many road links.

So, there getting that solution is not that easy. So, we take or we go with different you know heuristic methods to get the solution for user equilibrium or what would be the flow distribution?

What would be the time? All those we get through heuristic method. The first and the simplest one in this case is under this deterministic assignment is basically all-or-nothing assignment. So, that comes under deterministic assignment.

And this is the oldest simplest but yet I would say still it is interesting because of so many reasons. Although it is approximate many times it may not give you the right results or the character result. But still a to start with always we start with this all-or-nothing assignment.

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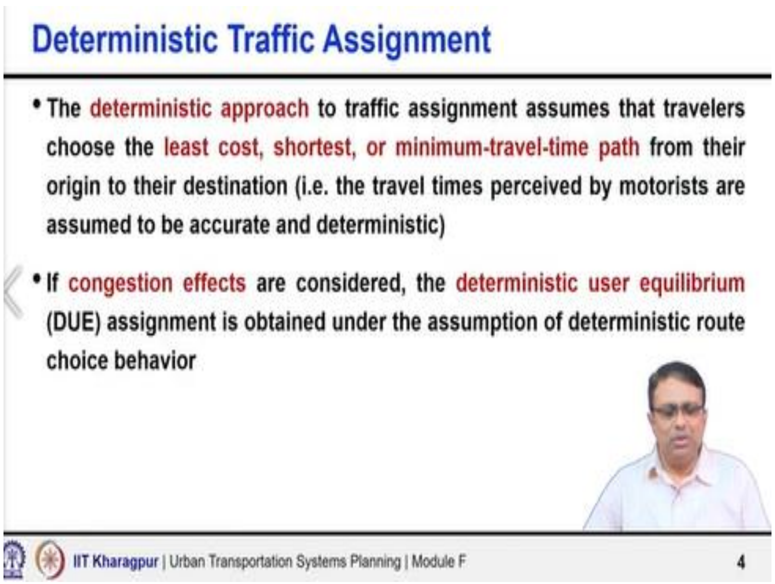

Now what we mean by this we shall discuss we shall also take an example. But let me first tell you little bit more about this deterministic traffic assignment. So, this deterministic approach to traffic assignment assumes that travelers choose the least cost, shortest or you can also call it the minimum travel path from their origin to their destination. And that means in a way that travel time perceived by motorists are assumed to be accurate.

This was a very, very fundamental assumption I mentioned it earlier. But once again I am repeating it that the travel time perceived by motorists are assumed to be accurate and deterministic. The, if you consider the congestion effect then the deterministic user equilibrium assignment is obtained under the assumption of deterministic route choice behaviour that means deterministic route choice behaviour what is deterministic?

Please recall our discussion about even the mode choice context, deterministic route choice behaviour means we know that the route choice is going to be this one. So, if we are doing based on travel time, that means everybody has exact understanding of the travel time. And you know they know exactly what is the real travel time no other error and no nobody considers any other factor all such kinds of assumptions are there.

And that is why we can say that you know everybody is taking the least cost path, or in a way that that is what it is that minimizing the his or her own utility. But in the deterministic framework, that means there is no erect term here as exactly the way we discussed it for the mode choice modelling, we said disaggregate choice model and then we said that deterministic disaggregate choice model and then subsequently introduced to you to the probabilistic choice model.

The probabilistic came because of the variation and because of the fact that things are not deterministic. So, here when we are talking about user equilibrium under this condition deterministic that means we are obtaining the user equilibrium with the assumption of deterministic route choice behaviour and as I said earlier also the travel time perceived by motorist to be accurate and deterministic. That is the again an important thing to remember.

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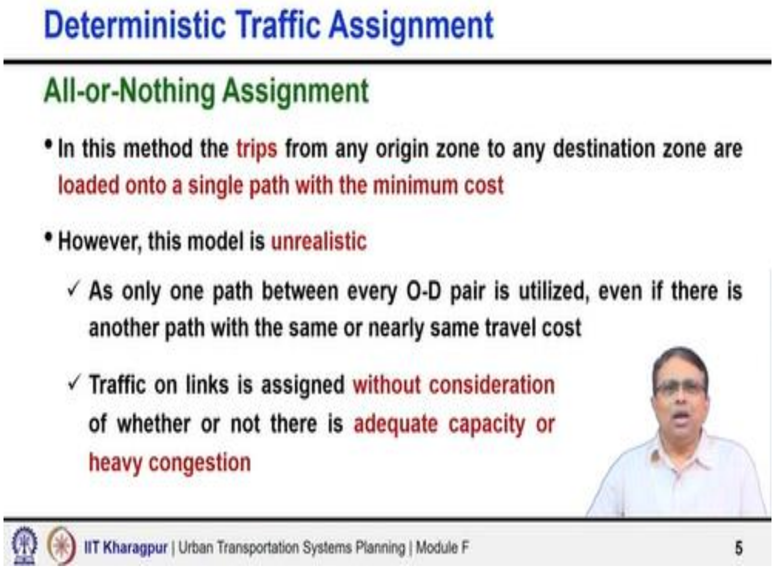

Now as I said the coming back to the all-or-nothing assignment, this is the simplest one. And in this method trips from any origin zone to any destination zone, are loaded onto the single path. And which is that single path? Single path is that one with the minimum cost. Now cost is a general representation of the deterrence if we are considering if you are doing the assignment considering the time is your deterrence, then it is the minimum travel time.

If you take really the actual operating cost then it is then minimum operating cost. If you consider generalized cost, it could be generalized cost, cost is a general representation of the deterrence. However, this model is unrealistic, why it is unrealistic? First, in this case only one path between every O-D pair is utilized, between one origin and one destination. There could be multiple path and urban network where this kind of assignment is primarily used.

And this is of course a course an urban transportation system planning where there are multiple paths. You think of any urban area in majority of the case of course in large urban area practically for all origin or destinations you will have multiple path even in a moderate town also mid-size town also even multiple path. So, we are actually doing our loading the whole demand only on one path.

So, even if there is another path or there are alternative paths, with the same or nearly central cost. The second traffic on link is assigned without consideration of whether or not there is adequate capacity or heavy congestion. Again, what does it mean? We are finding out what is the travel time? Now, what is this travel time? This may be the average travel time, this may be 0 flow travel time, free flow travel time when the road is empty there is very little traffic or practically no traffic.

Then how much time it takes that is the shortest path that is before loading the network, that is the network condition think of a network multiple load repairs and we have not yet loaded the traffic demand onto the network. So, without any demand or with it with very negligible demand around that situation, whatever is the travel time we know that and based on that we are finding out the shortest path and assign every demand between one origin and one destination along the shortest path only.

And not assigning any demand any other demand to any of the remaining alternative roots. Now the question is that if there is congestion and you find out that the shortest path is maybe 15 minutes. But after assigning the flow it may remain around 15 minutes that is again possibility. If you theoretically that is possible because if you consider that your road link is having a very high capacity.

And the loading whatever you are doing is not that significant. In that case the after assigning that flow which will be you know, almost not so significant or rather negligible still negligible as compared to the capacity of the path. Then the travel time may remain same or more or less same and still that path may be the shortest path. But in many cases urban context particularly if you see the scenario there is a huge imbalance between the demand and supply.

The networks are really the capacities inadequate or the demand at least we can say moderately that demand is almost near the capacity, I mean heavily loaded. So, under such condition of course there will be in some links the whatever I am assigning the road may not be able to take the demand. So, that means the demand if it may be even more than the capacity so the road link cannot take that much you know.

The other it could be it may be still not of to the level of capacity but close to capacity. So, that means there is a heavy congestion on road. So, in both cases the travel time is not going to be same as what we assumed initially before assigning traffic, maybe it was 15 minute and now it may be 30 minute, after assigning the load. And you may eventually find there is another route where the travel time is maybe 22 minute.

So, earlier the 22 minute was not at all considered because 15 minute path was the cheapest or was the with the least travel time. So, we assigned everything but it is no more 15 minute, it is actually now 30 minute. And 22 minute there is still a path another alternative path which is cheaper. So, obviously as long as there is another cheaper path, this is not the result is not realistic that is the point.

And also, as we said that we might be loading it with a demand, which is unrealistic in the sense that that link cannot take that much demand. So, everything is possible. So, if the demand is reasonably high where there is heavy congestion which will result into heavy congestion. Or the demand is even more than the capacity which is available for the path then by assignment is not going to give me the realistic result, unrealistic result I am going to get.

So, that is what we say that model is often realistic rather I should say specially in the urban context. And when we talk about the peak hour demand because in almost all Indian cities, they the peak hour demand is very significant as compared to that capacity. And in many cases may be the one may be even higher than for some of the links is higher than what is the available capacity.

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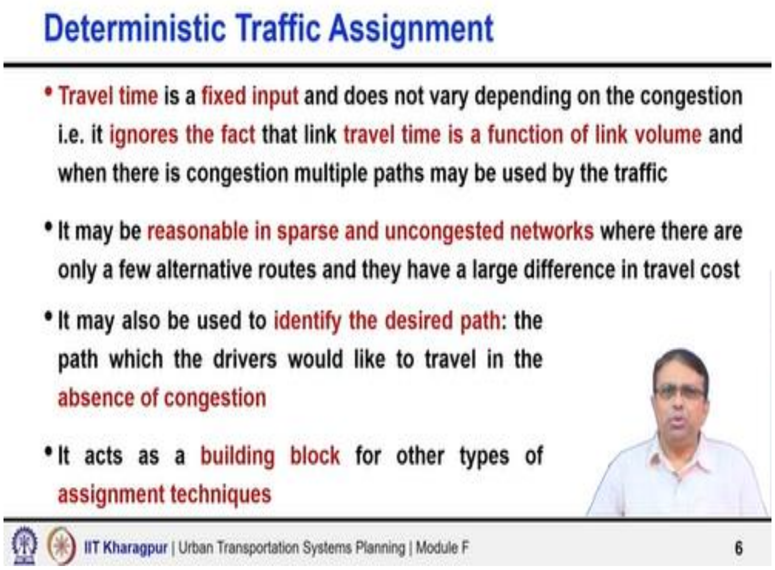

So, the as I said the travel time is a fixed input and, in this case, does not vary depending on the congestion because we are not rechecking we are simply calculating the shortest path and then you know, assigning the whole flow along the shortest path. So, that means it ignores the fact that the link travel time is a function of link volume and when there is congestion multiple paths may be actually used by the traffic rather than a single path.

Of course, every algorithm has its or every approach has its own application domain as well that also need to understand. So, when still this all-or-nothing assignment can give us good result, yes as I said earlier. If the assigned demand is negligible as compared to the capacity of the road links or the paths then it is fine. So, that 15 minute may still remain 15 minute or maybe 15 minute may become 16 or 17 minutes.

And still as long as even after assigning the demand, still the travel time is lower as compared to the alternative routes. This may happen under two conditions or multiple conditions rather but two conditions I can remember immediately where the demand is in significant very low has compared to the capacity. And there are second days that there are only a few routes and even if the shortest path actual travel time becomes higher.

But still it is lesser that means you have alternatives which have significantly large difference in travel time. And it is large enough, so that even if the present shortest path travel time changes still the path remains short shortest path. That means it may be that you have only a few alternatives and one may be if 15 minute and the next one may be 16 minute that kind of thing or maybe 70 minute or 45 minutes.

And in such that even if you load it, after loading also 15 may become 20 to 20 by become 25, but still it is cheaper or it is this shortest one. So, this kind of alternative scenarios can keep or the other thing that demand may be so insignificant as compared to the capacity. Maybe you can show it in the lean hour or peak hour. Urban areas still all other thing may occur or you can consider the generally the rural area. They are also you can have although our focus is urban transportation planning.

But in rural area also you can assign do the assignment of traffic, but in rural case the demand is not so high. So, it may happen that in regional level or rural level when you assign the traffic, even if there are alternative paths still the shortest path generally everybody will use because the demand is not significant. So, that is what we say that it may also be used the second part is also be used to identify the desired path. What it shows? Ideally what people would like to do?

So, it may be the best choice provided there is no congestion. So, we can say that the shortest path whatever I am getting now is almost like an ideal choice for people. If there is no congestion that had it been that you had enough capacity. The capacity is enough as compared to the demand then everyone travels along that path only and your solution could have been the correct one.

So, that something we can say is the extreme end like the what is the most ideal solution. But in reality, not everybody will be able to do that. And the solution will deviate from that kind of ideal solution if I consider it as a ideal for example. And they will deviate why? Because there is congestion. So, the travel time has increased, so it is lower shortest part because not every vehicle or all vehicle can be accommodated with the part because of the capacity constant.

So, it is something like one is the what is what is likely to happen in practice because of certain constant because the roads do not have adequate capacity because of congestion will build and all those. But otherwise the desired thing should have been that. Like if I put enough infrastructure and make the whole transportation network like that then everybody would love to use that path only.

So, that gives some kind of like where is the my base level or what is the best that is that probably people would have loved to you know experience. Also, as you will see subsequently that this all-or-nothing assignment it acts as a building block for other types of assignment techniques. So, we know that this is like this everybody would desire to use it if there is no congestion, no problem, no capacity constraints, but then we bring those things.

So, now the congestions, so now how we can improve my solution. So, it is most of this other algorithm are actually building or developed based on this further development further, you know, some kind of approximation some kind of consideration of the real-life constraint. So, this is something which can act as a building block for other types of assignment techniques developed subsequently.

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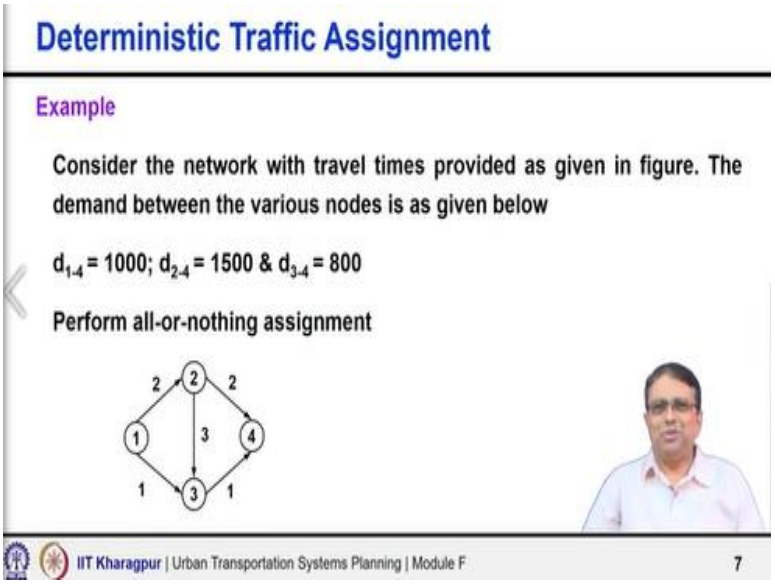

Now, with this background, I hope it somewhat clear to you. Let us take an example, it is a small example consider the network. So, I have got to 1, 2, 3, 4 nodes and if you connections the travel times are given and the demands are given like 1 to 4 1000, 2 to 4 1500 and 3 to 4 800. Now what we are trying to do here? We are trying to perform all-or-nothing assigned when that means we are trying to assign this demands.

Following all-or-nothing traffic assignment technique. And trying to see then what will be my distribution of demand or how much development will come to each and every link.

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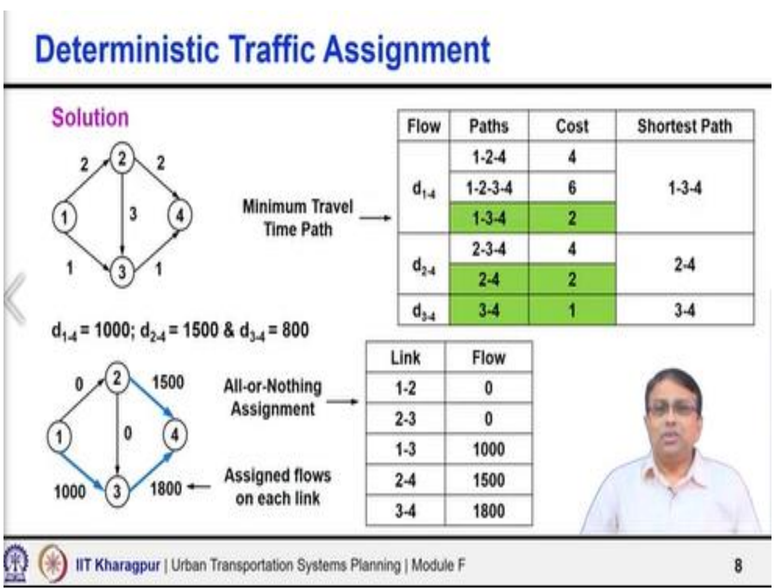

So, the first step to do that, I need to find out the shortest path. So, let us take 1 to 4, since this is a small network and very simple example. So, we have not applied any formal algorithm here to get the shortest path like formal algorithm means what you have learned you could you know in a real network you could use Dijkstra's algorithms, you could fly itself with them both this we have covered there are also other algorithms, of course.

So, you could use this algorithm but here we have not used any algorithm we simply find out you because it is a simple network. So, you can see that there are three paths which are actually connecting 1 to 4th with you know directional movement. So, one is 1 to 4 the cost is 4 and then another is you go from 1 to 2, 2 to 3, 3 to 4. So, that cost is 6, and another is you can travel from 1 to 3 and then 3 to 4, so that the cost is 1 + 1 2.

So, what is the least cost path or the minimum cost path is the path 1, 3, 4. So, for 1 to 4, 1, 3 4 is the least cost path. Similarly, for 2 to 4 we know that there are 2 paths 1 is 2 to 3 and 3 to 4 with the cost of 4 and then 2 to 4 directly with the cost of 2. So, obviously 2 to 4 is the minimum and 3 to 4 is just one connection. So, there is no alternative so because the other movements you see the arrow head. So, it shows the directional part.

So, like the 2 to 3 you can travel but you cannot travel from 3 to 2 because the arrowhead is only towards 3. So, you have only one path and that is 3 to 4, so what I will do now? Now, I will say 1 to 4, how much is the demand? 1 to 4 demand is 1000. So, all 1000 I will assign 1 to 4, so my 1 to 3 becomes 1000, 3 to 4 also becomes 1000 then the second one 2 to 4 directly to 2 to 4. I will assign then 1500 and 3 to 4, I will assign 1800.

So, what we will get? You will get the flow like this 1 to 2 0, 2 to 3 0, 1 to 3 1000 2 to 4 1500 and 3 to 4 1000 + 800 so it is 1800. So, that is what gives you the deterministic assignment flow.

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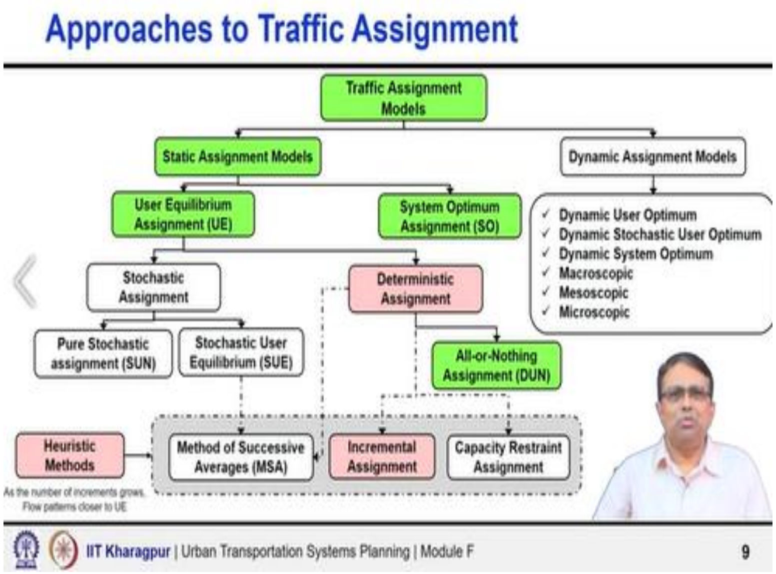

Now we go to the next one that is called incremental assignment.

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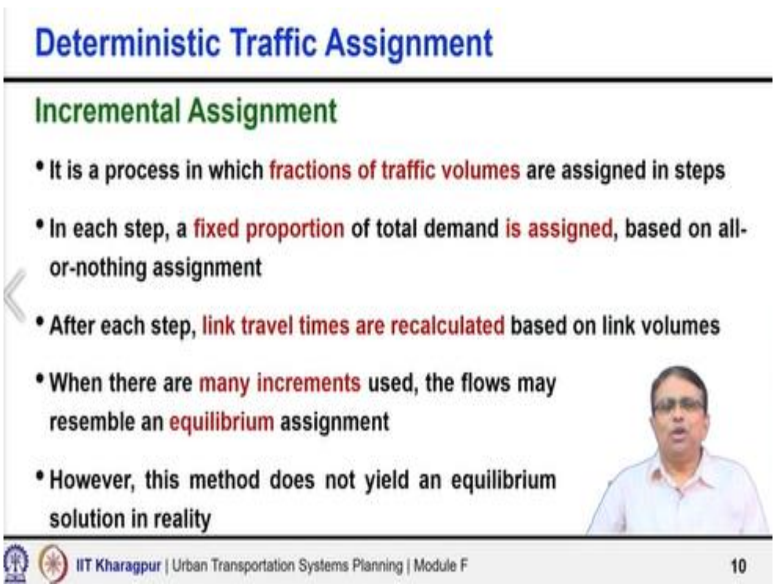

Now what is incremental? Incremental is we know the problem what is really the problem with the all-or-nothing assignment. Now till me what is the basic problem all-or-nothing assignment is not so bad, except the problem is that we really do not know after assigning the whole demand along the shortest path whether that path is still the shortest path or not. And many cases we know that the deviation may be too much;

Because the whole demand may be huge demand has compared to the capacity and once, I assign then maybe my whole shortest path definition will be lost. That it is no more going to be a shortest path. So, but we understand that is people do like to travel along the shortest path only thing the problem is because of assigning too much if at all the load is too much then assigning the too much of load at a time will bring a different shortest path altogether.

That part which was considered to be the shortest path will no more remain as the shortest path. That is the basic problem. So, people try to use incremental assignment to solve it. What will simply do? Do not load the complete demand at a time. So, what we try to do all these are explained here. Suppose you decide that I will load it in four different steps how many steps that up to you.

You can make it 4 you can make it even 14. So, but then the total demand is distributed into or divided into this many number of increments and suppose I make five increments so instead of loading the complete demand in one go. I will load equal amount that means 20% demand each time and five times I will load it. But with a small change, what I will do? I will load first 20 % based on zero flow travel time. That is what you know till that time. But then after assigning to 20% I will recalculate and see what is my shortest path.

Then next 20% I will load along the present shortest path that means after assigning 20% whatever is the shortest path now, next 20% will be assigned on that shortest part along the shortest path. I will again recalculate see what is my shortest path now next 20% I will load it again along the shortest part. That is what it is. So, in this case if my number of increments is really high then theoretically, I would be very close to the actual solution.

Because my travel time is not changing drastically small, small incremental loading and accordingly the change, fantastic. It can give good result when there you consider only one origin destination pair. But if you consider it realistic network large network, you cannot you know, do the increment really you know with a very small loading. So, maybe you will do it in 4, 5, 6, 10 loadings at best or maybe little even more.

But that also for a large network and with all or the pairs it will not guarantee you that after the last incremental loading your travel time on all alternative paths for a particular origin and the particular destination will still be equal. It may be that before that it was fine, but after the last incremental loading you have loaded it almost near capacity. So, there will be a sharp jump in the travel time, it may happen all this kind of thing.

So, it does not guarantee you that you will get still exactly an equilibrium in reality. But what it can give you definitely a solution, which is much better than all-or-nothing. But understand that here also every after every recalculation before we load the incremental loading or before we actually load the network with the next round of loading, we are still assigning it as per the on all-or-nothing assignment. But only thing not the whole demand at a time.

We are doing it incrementally so that the travel time you know, as far as possible we are trying to make it realistic. So, the solution has to be better and definitely it will be better. But still no guarantee that it will be exactly the user optimal not that time on all alternative path will exactly be same which are all used by the traffic. But of course, probably you would find that earlier it was 50 minute and the next you still have all flow along the 50 minute route. And then no flow on the minute road because you started with 15 or 10 probably.

Now you will get not so much of difference maybe we will find some 20 some route 30 some 25, so the difference will be less. But it again depends on how much incremental loading how many increments you already doing. So, the solution will be no doubt better but still the user equilibrium solution, it is still not guaranteed.

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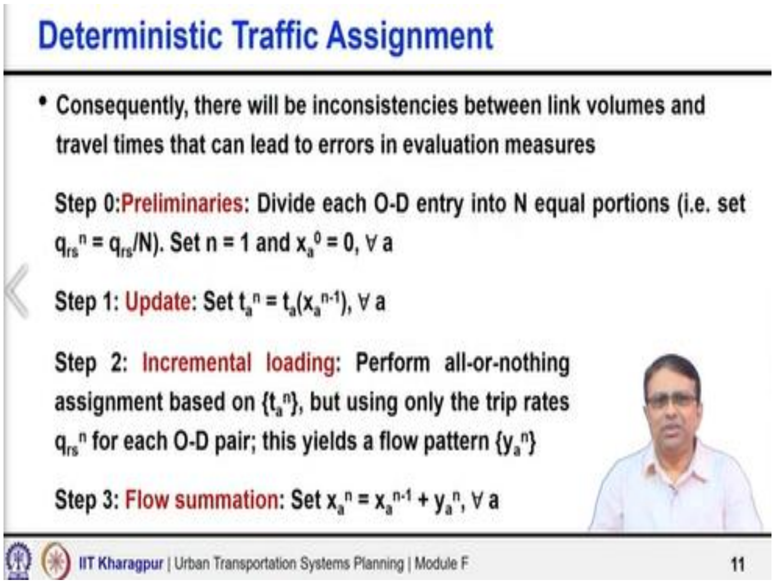

So, here I have mentioned that stepwise you divide the whole load into a number of equal thing and then set the load every time. That much amount after you then recalculate the travel time and then based on this travel time you do the next loading following all-or-nothing assignment. Then add up the flow and see now total flow is how much.

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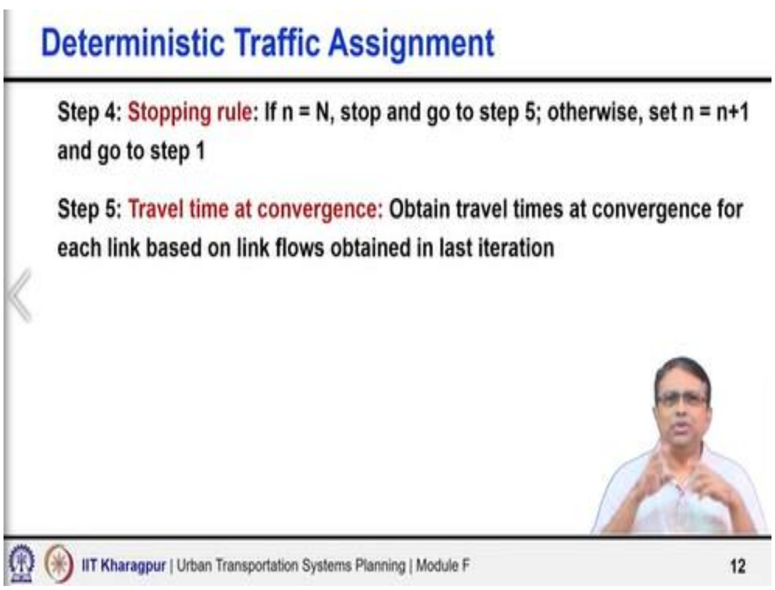

And then if it is n, the number of iterations what you wanted to do then you directly go and see then what is my final travel time. And if that has not been done then go back to step 1. That means again you calculate now the travel time and accordingly assign the next incremental load.

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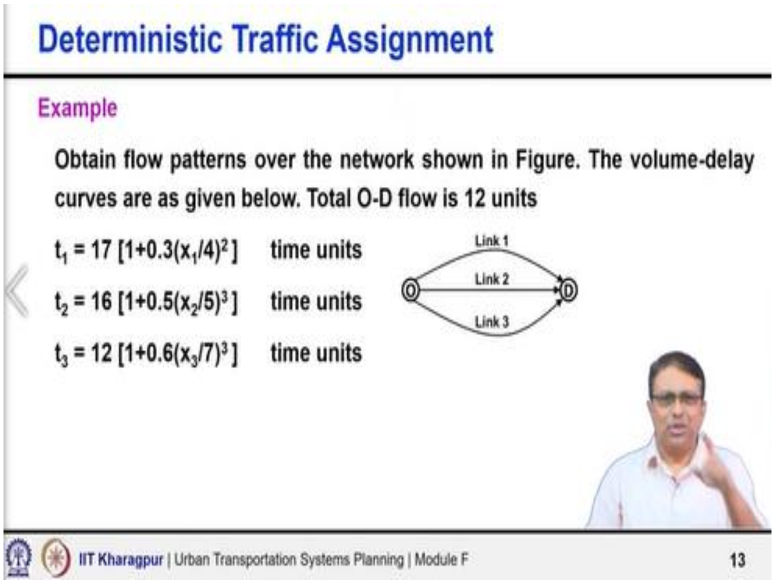

So, small example is there, there are 3 paths link 1, link 2 and link 3. And we want to assign 12 units following incremental loading. And we want to use 4 increments in this case.

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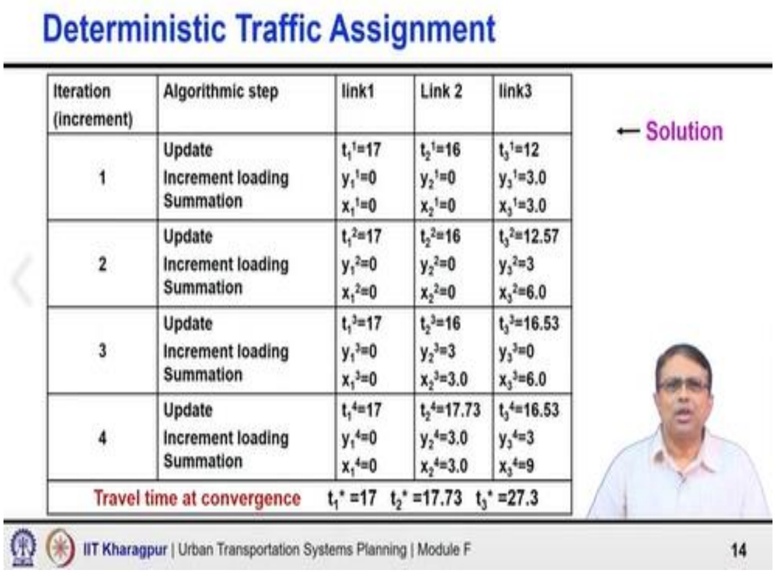

So, what we did 4 means each case, we will assign 3 units. So, I have first calculated the travel time as 17, 16, 12, so link 3 is the cheapest. So, the 3 unit is incremental loading is done along that remaining 0, 0. And since this is the first iteration, so the summation or the total load also is same. Now after this one I go to step 2, I update, update the travel time. Now, I find that 12-minutes become 12.57, but still that is the cheaper.

So, the next incremental loading is also done along path 2. And I calculate the summation what you can see 2 times incremental loading has gone to link 3 because link 3 is still remain shortest path in both iterations. So, 6 unit is the total flow, now we go to iteration 3 with 6-unit flow now I took calculate the travel time and I find it 16.53, which is higher than 16. So, now my link 2 becomes the cheaper.

So, the next 3 unit incremental load is the sign to this shortest path, which is linked 2. So, with this the cumulative load becomes 0 on link 1, 3 on link 2 and 6 on link 3. Now with these the travel time of 1 remaining change because no load has been assigned to a still remain same all throughout iterations up to this. But path to travel time becomes 17.73 and third route that is the link 3 remainder 16.53. So, now my shortest path is basically again link 3.

So, then next incremental load is also done on link 3. So, my total flow distribution becomes 0, 3 and 9 along link 1, link 2 and link 3 respectively. Interestingly if you see till the iteration just before the loading 4th loading you see the travel times as 17, 17.73 and 16.53 quite good. But the last incremental loading has to distort it the whole thing. So, now you have 17 minute you have 17.73, but t 3 or the link 3 travel time becomes 27.3 which is significantly different.

So, you might have instead of the same problem you can try to solve by all-or-nothing assignment you will find that t 3 could have been much higher maybe 70, 80, I do not know. Something would have come but much higher compared to that now it is a better solution. Because it little bit redistributes the flow and then you know somewhere link 2 also got a share. So, it is a better solution but still it does not guarantee the user optimal solution.

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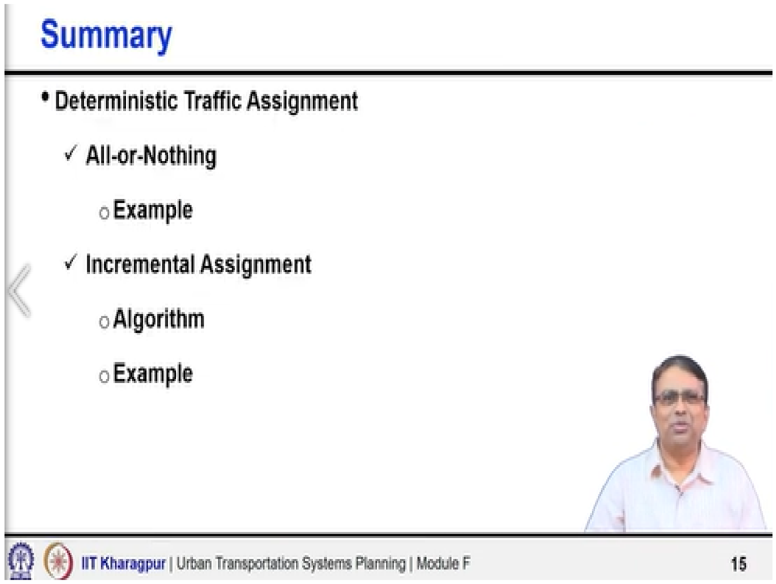

So, what we discussed altogether if I have to summarize. We discussed about the deterministic traffic assignment technique told you what is actually the meaning of deterministic traffic assignment. Then we first introduced to you the all-or-nothing assignment technique with an example told you where it may work where it may not work, in congested urban network it is not going to give you very well solution.

Then to overcome that what we do a further improvement is the incremental assignment. So, explained you what you do in incremental assignment and they do an example show you the application how you can improve your result, but yet to conclude that this still may not give you the true user equilibrium distribution of flow. So, with this I close this lecture we will continue in the next lecture, thank you so much.